

Contents

Introduction

xiii

I	A First Course in Network Calculus	1
1	Network Calculus	3
1.1	Models for Data Flows	4
1.1.1	Cumulative Functions, Discrete Time versus Continuous Time Models	4
1.1.2	Backlog and Virtual Delay	6
1.1.3	Example: The Payout Buffer	7
1.2	Arrival Curves	8
1.2.1	Definition of an Arrival Curve	8
1.2.2	Leaky Bucket and Generic Cell Rate Algorithm	12
1.2.3	Sub-additivity and Arrival Curves	17
1.2.4	Minimum Arrival Curve	20
1.3	Service Curves	22
1.3.1	Definition of Service Curve	22
1.3.2	Classical Service Curve Examples	25
1.4	Network Calculus Basics	28
1.4.1	Three Bounds	28
1.4.2	Are the Bounds Tight ?	33
1.4.3	Concatenation	34
1.4.4	Improvement of Backlog Bounds	36
1.5	Greedy Shapers	37
1.5.1	Definitions	37
1.5.2	Input-Output Characterization of Greedy Shapers	38
1.5.3	Properties of Greedy Shapers	40
1.6	Maximum Service Curve, Variable and Fixed Delay	42
1.6.1	Maximum Service Curves	42
1.6.2	Delay from Backlog	45
1.6.3	Variable versus Fixed Delay	48
1.7	Handling Variable Length Packets	49

1.7.1	An Example of Irregularity Introduced by Variable Length Packets	49
1.7.2	The Packetizer	50
1.7.3	A Relation between Greedy Shaper and Packetizer	55
1.7.4	Packetized Greedy Shaper	58
1.8	Lossless Effective Bandwidth and Equivalent Capacity	64
1.8.1	Effective Bandwidth of a Flow	64
1.8.2	Equivalent Capacity	65
1.8.3	Example: Acceptance Region for a FIFO Multiplexer	67
1.9	Proof of Theorem 1.4.5	68
1.10	Bibliographic Notes	71
1.11	Exercises	72
2	Application of Network Calculus to the Internet	83
2.1	GPS and Guaranteed Rate Schedulers	83
2.1.1	Packet Scheduling	83
2.1.2	GPS and a Practical Implementation (PGPS)	84
2.1.3	Guaranteed Rate Schedulers	86
2.2	The Integrated Services Model of the IETF	89
2.2.1	The Guaranteed Service	89
2.2.2	The Integrated Services Model for Internet Routers	89
2.2.3	Reservation Setup with RSVP	90
2.2.4	A Flow Setup Algorithm	93
2.2.5	Multicast Flows	94
2.2.6	Flow Setup with ATM	94
2.3	Schedulability	94
2.3.1	EDF Schedulers	95
2.3.2	SCED Schedulers [65]	97
2.3.3	Buffer Requirements	103
2.4	Application to Differentiated Services	104
2.4.1	Differentiated Services	104
2.4.2	A Bounding Method for Aggregate Scheduling	104
2.4.3	An Explicit Delay Bound for Differentiated Services Networks	110
2.4.4	Bounds for Aggregate Scheduling with Dampers	117
2.5	Exercises	121
II	Mathematical Background	123
3	Basic Min-plus and Max-plus Calculus	125
3.1	Min-plus Calculus	125
3.1.1	Infimum and Minimum	125
3.1.2	Dioid $(\mathbb{R} \cup \{+\infty\}, \wedge, +)$	127
3.1.3	A Catalog of Wide-sense Increasing Functions	128

3.1.4	Pseudo-inverse of Wide-sense Increasing Functions	129
3.1.5	Concave, Convex and Star-shaped Functions	132
3.1.6	Min-plus Convolution	133
3.1.7	Sub-additive Functions	140
3.1.8	Sub-additive Closure	143
3.1.9	Min-plus Deconvolution	147
3.1.10	Representation of Min-plus Deconvolution by Time Inversion	151
3.1.11	Vertical and Horizontal Deviations	154
3.2	Max-plus Calculus	155
3.2.1	Max-plus Convolution and Deconvolution	155
3.2.2	Linearity of Min-plus Deconvolution in Max-plus Algebra .	156
3.3	Exercises	156
4	Min-plus and Max-plus System Theory	159
4.1	Min-plus and Max-plus Operators	159
4.1.1	Vector Notations	159
4.1.2	Operators	161
4.1.3	A Catalog of Operators	162
4.1.4	Upper and Lower Semi-continuous Operators	163
4.1.5	Isotone Operators	164
4.1.6	Linear Operators	165
4.1.7	Causal Operators	169
4.1.8	Shift-invariant Operators	170
4.1.9	Idempotent Operators	171
4.2	Closure of an Operator	171
4.3	Fixed Point Equation (Space Method)	175
4.3.1	Main Theorem	175
4.3.2	Examples of Application	177
4.4	Fixed Point Equation (Time Method)	181
4.5	Conclusion	182
III	A Second Course in Network Calculus	183
5	Optimal Multimedia Smoothing	185
5.1	Problem Setting	185
5.2	Constraints Imposed by Lossless Smoothing	187
5.3	Minimal Requirements on Delays and Playback Buffer	188
5.4	Optimal Smoothing Strategies	189
5.4.1	Maximal Solution	189
5.4.2	Minimal Solution	189
5.4.3	Set of Optimal Solutions	190
5.5	Optimal Constant Rate Smoothing	191
5.6	Optimal Smoothing versus Greedy Shaping	195

5.7	Comparison with Delay Equalization	199
5.8	Lossless Smoothing over Two Networks	201
5.8.1	Minimal Requirements on the Delays and Buffer Sizes for Two Networks	202
5.8.2	Optimal Constant Rate Smoothing over Two Networks	204
5.9	Bibliographic Notes	206
6	FIFO Systems and Aggregate Scheduling	207
6.1	Introduction	207
6.2	General Bounds for Aggregate Scheduling	208
6.3	Stability of a Network with Aggregate Scheduling	209
6.3.1	The Open Issue of Stability	209
6.3.2	The Ring is Stable	210
6.4	Bounds for a FIFO Service Curve Element	214
6.5	Bounds for a Network of FIFO CBR Servers	218
6.5.1	Closed Form Bounds for an ATM Network with Strong Source Rate Conditions	218
6.5.2	Proof of Theorem 6.5.1	220
6.6	Bibliographic Notes	224
6.7	Exercises	225
7	Adaptive and Packet Scale Rate Guarantees	227
7.1	Introduction	227
7.2	Adaptive Guarantee	227
7.2.1	Limitations of the Service Curve Abstraction	227
7.2.2	Definition of Adaptive Guarantee	228
7.2.3	Properties of Adaptive Guarantees	230
7.3	Application to the Internet: Packet Scale Rate Guarantee	234
7.3.1	Definition of Packet Scale Rate Guarantee	234
7.3.2	Practical Realization of Packet Scale Rate Guarantee	236
7.3.3	Proof of Theorem 7.3.1	238
7.4	Bibliographic Notes	241
7.5	Exercises	241
8	Time Varying Shapers	243
8.1	Introduction	243
8.2	Time Varying Shapers	243
8.3	Time Invariant Shaper with Initial Conditions	245
8.3.1	Shaper with Non-empty Initial Buffer	245
8.3.2	Leaky Bucket Shapers with Non-zero Initial Bucket Level .	246
8.4	Time Varying Leaky-Bucket Shaper	248
8.5	Bibliographic Notes	249

9 Systems with Losses	251
9.1 A Representation Formula for Losses	251
9.1.1 Losses in a Finite Storage Element	251
9.1.2 Losses in a Bounded Delay Element	253
9.2 Application 1: Bound on Loss Rate	255
9.3 Application 2: Bound on Losses in Complex Systems	256
9.3.1 Bound on Losses by Segregation between Buffer and Policer	256
9.3.2 Bound on Losses in a VBR Shaper	259
9.4 Solution to Skohorkhod's Reflection Problem with Two Boundaries	261
9.5 Bibliographic Notes	265
Bibliography	267
Index	273