

Contents

Foreword — VII

Preface — IX

Part I: *Mathematica* and ordinary differential equations

1	Certain theoretical results — 5
1.1	Existence and uniqueness theorems — 5
1.2	Exponential, logarithmic, and trigonometric functions — 6
1.2.1	Exponential and logarithmic functions — 6
1.2.2	Trigonometric functions — 9
1.2.3	Gaussian, error, and sign functions — 15
1.2.4	Sign, unit step, clip functions — 16
1.2.5	Dirac delta functions — 17
1.2.6	Gradient, divergence, and Laplacian — 18
1.3	Gamma and beta functions — 20
1.3.1	Gamma function — 20
1.3.2	Beta function — 21
1.4	Hypergeometric equation — 23
1.4.1	Hypergeometric functions — 23
1.4.2	Hypergeometric equation — 24
1.5	Hyperbolic functions — 24
1.6	Bessel functions — 26
1.6.1	Modified Bessel functions — 28
1.7	Legendre functions — 29
1.8	Jacobi polynomials — 31
1.9	Mathieu functions — 32
1.9.1	Mathieu equation — 32
1.9.2	Mathieu functions — 35
1.10	Elliptic integrals, elliptic functions — 35
1.10.1	Elliptic integrals — 36
1.10.2	Elliptic functions — 37
2	Elementary ordinary differential equations — 39
2.1	Classification of certain ordinary differential equations — 39
2.2	Planar phase portrait — 49
2.3	Solid phase portrait — 53
2.4	Straight integration of first-order derivatives — 57

- 2.5 Differential equations with separable variables — **60**
- 2.6 Homogeneous differential equations — **61**

3 First-order ordinary differential equations — 66

- 3.1 First-order linear differential equation — **66**
- 3.2 First-order inverse linear differential equation — **69**
- 3.3 Bernoulli differential equation — **71**
- 3.4 Riccati differential equation — **75**
- 3.5 Exact first-order ordinary differential equation — **82**
- 3.6 Lagrange differential equation — **88**
- 3.7 First-order implicit differential equation — **94**
- 3.8 Other first-order differential equations — **101**

4 Higher order and systems of ordinary differential equations — 104

- 4.1 Second-order linear differential equations — **104**
- 4.2 Bessel differential equation — **119**
- 4.3 Legendre differential equation — **123**
- 4.4 Mathieu differential equation — **126**
- 4.5 Equations with discontinuous coefficients or right-hand sides — **132**
- 4.6 Second-order nonlinear differential equations — **135**
- 4.7 Other higher-order differential equations — **146**
- 4.8 Systems of differential equations — **152**

Part II: ***Mathematica* and partial differential equations**

5 First-order partial differential equations — 171

- 5.1 First-order linear partial differential equations — **171**
- 5.2 First-order quasilinear partial differential equations — **187**
- 5.3 First-order nonlinear partial differential equations — **192**

6 Linear hyperbolic partial differential equations — 202

- 6.1 Hyperbolic equations with constant coefficients — **202**
- 6.2 Hyperbolic equations with variable coefficients — **225**
- 6.3 Hyperbolic equations on curvilinear domains — **234**
- 6.4 Hyperbolic equations in solid space — **246**
- 6.5 Klein–Gordon equation — **254**

7 Nonlinear and higher-order hyperbolic equations — 260

- 7.1 Sine-Gordon equation — **260**
- 7.2 Modified Klein–Gordon equation — **274**
- 7.3 Double sine-Gordon equation — **275**

7.4	Nonlinear hyperbolic equations —	277
7.5	Nonlinear hyperbolic equations in higher dimension —	283
8	Elliptic partial differential equations —	285
8.1	Harmonic functions —	285
8.2	Laplace equations on rectangles —	289
8.3	Laplace equations on arbitrary domains —	299
8.4	Poisson equations on rectangles —	310
8.5	Poisson equations on arbitrary domains —	311
8.6	Laplace equation in higher dimension —	317
8.7	Poisson equations in higher dimensions —	320
9	Parabolic partial differential equations —	323
9.1	1D homogeneous parabolic equations —	323
9.2	1D inhomogeneous parabolic equations —	332
9.3	2D homogeneous parabolic equations —	335
9.4	2D inhomogeneous parabolic equations —	341
9.5	The Burgers' equations —	344
9.5.1	The Burgers' equation in 1D —	344
9.5.2	The Burgers' equation in 2D —	349
9.6	Ansatz methods —	349
9.6.1	The tanh function method —	350
9.6.2	The Hirota bilinear method —	351
9.6.3	The exp function method —	353
9.6.4	The sine-cosine functions method —	354
9.7	The Fisher equation —	354
9.7.1	The Fisher equation without convection term by <i>Mathematica</i> —	354
9.7.2	The Fisher equation with convection term by <i>Mathematica</i> —	357
9.7.3	The Fisher equation with a convection term by the tanh method —	358
9.7.4	The generalized Fisher equation by the tanh method —	361
9.7.5	The Fisher equation with convection by the Hirota method —	372
9.8	The Fitzhugh–Nagumo and the Calogero equations —	375
9.8.1	The Fitzhugh–Nagumo equation —	375
9.8.2	The Calogero equation —	379
9.9	Modified Klein–Gordon equation by ansatz methods —	380
9.10	The double sine-Gordon equation by the tanh function method —	385
9.11	Continuous dependence on a parameter —	389
10	Third- and higher-order nonlinear partial differential equations —	393
10.1	The Korteweg–de Vries equations —	393
10.1.1	Algebraic solitons —	399
10.1.2	Soliton solutions to the Korteweg–de Vries equation —	403

10.1.3	Modified Korteweg–de Vries equation —	413
10.1.4	Potential Korteweg–de Vries equation —	415
10.1.5	The generalized Kudryashov method —	416
10.2	The Dodd–Bullough–Mikhailov equation —	420
10.3	The Tzitzeica–Dodd–Bullough equation —	426
10.4	The modified Kawahara equation —	431
10.5	The Benjamin equation —	434
10.6	The Kadomtsev–Petviashvili equation —	438
10.6.1	Single soliton solution —	439
10.6.2	Hirota bilinear method to the KPI equation —	444
10.7	The Sawada–Kotera equation —	446
10.8	The Kaup–Kupershmidt equation —	451

Bibliography — 459

Index — 463