

Contents

Foreword	v
I Formation of Singularities in the Mean Curvature Flow	
<i>Carlo Sinestrari</i>	1
1 Introduction	3
2 Geometry of hypersurfaces	5
3 Examples	9
4 Local existence and formation of singularities	10
5 Invariance properties	16
6 Singular behaviour of convex surfaces	20
7 Convexity estimates	23
8 Rescaling near a singularity	25
9 Huisken's monotonicity formula	28
10 Cylindrical and gradient estimates	32
11 Mean curvature flow with surgeries	36
Bibliography	39
II Geometric Flows, Isoperimetric Inequalities and Hyperbolic Geometry	
<i>Manuel Ritoré</i>	45
Preface	49
1 The classical isoperimetric inequality in Euclidean space	53
1.1 Introduction	53
1.2 Preliminaries	53
1.2.1 Area and volume	53
1.2.2 Variational formulas	55
1.2.3 The isoperimetric profile	56
1.2.4 Isoperimetric regions	57

1.3	The isoperimetric inequality in Euclidean space	58
1.3.1	A calibration argument	59
1.3.2	Schwarz symmetrization	61
1.3.3	Another proof of the isoperimetric inequality	66
2	Surfaces	69
2.1	Introduction	69
2.1.1	The Gauss–Bonnet Theorem	70
2.2	Curve shortening flow	72
2.2.1	Basic results	72
2.2.2	The avoidance principle	74
2.3	Applications of curve shortening flow to isoperimetric inequalities .	75
2.3.1	Planes	75
2.3.2	Spheres	82
3	Higher dimensions	85
3.1	Introduction	85
3.2	H^k -flows and isoperimetric inequalities	86
3.3	Estimates on the Willmore functional and isoperimetric inequalities	89
3.3.1	Euclidean spaces	89
3.3.2	3-dimensional Hadamard manifolds	91
3.4	Singularities in the volume-preserving mean curvature flow	96
4	Some applications to hyperbolic geometry	99
4.1	Introduction	99
4.2	Bounds on the Heegaard genus of a hyperbolic manifold	100
4.3	The isoperimetric profile for small volumes	102
	Bibliography	105