# Contents


**1 Lie methods in optics: an assessment**
PETER W. HAWKES 1
1.1 The arrival of Lie methods on the optical scene; a qualitative survey . . . . . 1
1.2 Pre-Lie and post-Lie . . . . . 4
1.3 Concluding remarks . . . . . 12
1.4 References . . . . . 13

**2 Holographic image processing, coherent optical computing, and neural computer architecture for pattern recognition**
WALTER SCHEMPP 19
2.1 Introduction . . . . . 20
2.2 Sequential data compression . . . . . 21
2.3 Applications: CD-A, CD-ROM and CD-E . . . . . 23
2.4 Parallel two-dimensional data compression . . . . . 27
2.5 The holographic geometry is sub-Riemannian . . . . . 29
2.6 Holographic reciprocity and coupling . . . . . 30
2.7 Elementary holograms and complete bipartite graphs . . . . 31
2.8 Holographic invariants and linear optical phase conjugation 32
2.9 Cascaded acousto-optic real-time kernel implementation . . 33
2.10 Classification of pixel mappings and holographic interferometry . . . . . 35
2.11 Non-linear real-time optical phase conjugation . . . . . 38
2.12 The classical SAR processing architecture . . . . . 38
2.13 Neural computer architecture for pattern recognition . . . . 40
2.14 Conclusions . . . . . 42
2.15 Acknowledgements . . . . . 43
2.16 References . . . . . 43

**3 Canonical integration and analysis of periodic maps using non-standard analysis and Lie methods**
ETIENNE FOREST AND MARTIN BERZ 47
3.1 Introduction: The equation of motion . . . . . 47



Bibliografische Informationen
http://d-nb.info/891684816

digitalisiert durch DEUTSCHE NATIONAL BIBLIOTHEK

3.2 Conventional approach for the study of $\mathcal{G}(s; s+1)$ . . . . . 48
3.2.1 Ray tracing . . . . . 48
3.2.2 Normalization of $H(t)$ . . . . . 49
3.2.3 Conclusion on old methods . . . . . 49
3.3 A new approach for the study of $\mathcal{G}(s; s+1)$ . . . . . 50
3.4 Canonical integration in the symplectic group . . . . . 52
3.4.1 Explicit integration in a Lie group . . . . . 52
3.4.2 Implicit integration in a Lie group . . . . . 53
3.4.3 Conclusion . . . . . 54
3.5 Non-standard analysis and its application to map extraction 54
3.6 Normal form procedures on a power series map . . . . . 58
3.6.1 A first order calculation on the Lie representation . 58
3.6.2 Conclusion: to higher order with the differential algebra tools . . . . . 60
3.7 The Floquet representation and its Hamiltonian-free description . . . . . 61
3.7.1 The old way: normalizing the hamiltonian . . . . . 61
3.7.2 Conclusion on Hamiltonian normalization . . . . . 62
3.7.3 Floquet transformation on the map . . . . . 62
3.8 Acknowledgements . . . . . 64
3.9 References . . . . . 64

**4 Concatenation of Lie Algebraic Maps**
LIAM M. HEALY AND ALEX J. DRAGT 67
4.1 Introduction . . . . . 67
4.1.1 Definitions and terminology . . . . . 68
4.1.2 The task of concatenation . . . . . 69
4.2 Ideal structure of the Lie algebra . . . . . 70
4.2.1 Homogeneous products . . . . . 70
4.2.2 Inhomogeneous products . . . . . 73
4.3 Lie algebraic tools . . . . . 76
4.3.1 The exchange rule . . . . . 76
4.3.2 Combining transformations and factoring a single transformation . . . . . 77
4.4 Computation method for the concatenation formula . . . . 79
4.4.1 Putting a pair of transformations into standard factorization . . . . . 80
4.4.2 Moving the first-order term . . . . . 80
4.4.3 Moving second-order terms . . . . . 84
4.4.4 Moving higher-order terms . . . . . 85
4.5 Summary . . . . . 87
4.A A basis for the Lie algebra . . . . . 88
4.B Concatenation formulae for $N = 6$ . . . . . 90
4.C The results of the separation procedure for $N = 6$ . . . . . 93
4.6 Acknowledgements . . . . . 94

4.7 References . . . . . 94

**5 Dispersion-diffraction coupling in anisotropic media and ambiguity function generation**
MOSHE NAZARATHY, JOSEPH W. GOODMAN, AND MARK KAUDERER 97
5.1 Introduction . . . . . 97
5.2 Dispersion relations in linear homogeneous media . . . . . 98
5.3 Plane wave spectrum representation . . . . . 101
5.4 Canonical operator formulation . . . . . 103
5.5 Uncoupling of anisotropy and dispersion . . . . . 103
5.6 Diffraction-dispersion analogy . . . . . 105
5.7 Diffraction-dispersion interaction . . . . . 106
5.8 Space-time duality . . . . . 109
5.9 Discussion . . . . . 110
5.A Appendix: Canonical operator algebra . . . . . 111
5.A.1 Basic operator definitions . . . . . 111
5.A.2 Notation of variables . . . . . 111
5.A.3 Operator algebra . . . . . 111
5.10 References . . . . . 113

**6 Elements of Euclidean optics**
KURT BERNARDO WOLF 115
6.1 Introduction . . . . . 115
6.2 The bundle of rays in geometric optics . . . . . 118
6.3 Lie operators on the Euclidean group . . . . . 119
6.4 Generators of the Euclidean group . . . . . 122
6.5 Coset spaces and rays . . . . . 124
6.6 Euclidean group action on rays in geometric optics . . . . . 126
6.7 The Euclidean algebra generators on rays . . . . . 128
6.8 The coset space of wavefront optics . . . . . 131
6.9 Helmholtz optics . . . . . 133
6.10 The Hilbert space for Helmholtz optics . . . . . 135
6.11 The Euclidean algebra generators in Helmholtz optics —wavization . . . . . 140
6.12 The ray direction sphere under Lorentz boost transformations . . . . . 144
6.13 Relativistic coma in geometric optics . . . . . 146
6.14 Relativistic coma in Helmholtz wave optics . . . . . 149
6.15 Reflection, refraction, and concluding remarks . . . . . 154
6.16 Acknowledgements . . . . . 160
6.17 References . . . . . 160

**7 The map between Heisenberg-Weyl and Euclidean optics is comatic**
VLADIMIR I. MAN'KO AND KURT BERNARDO WOLF 163
7.1 Introduction . . . . . 163
7.2 From Snell's law to the Hamilton equations . . . . . 165
7.3 The paraxial régime and Heisenberg–Weyl optics . . . . . 168
7.4 The opening coma map . . . . . 170
7.5 The map between Heisenberg-Weyl and Euclidean free rays 173
7.6 The symplectic group on Euclidean phase space . . . . . 175
7.7 The Euclidean and Lorentz groups on Heisenberg-Weyl phase space . . . . . 178
7.8 Spherical aberration, coma, and point transformations in phase space . . . . . 183
7.9 The Hilbert spaces for Heisenberg-Weyl and Euclidean optics . . . . . 185
7.10 Plane waves and the coma kernel . . . . . 187
7.11 Gaussians, non-diffracting beams, and concluding remarks . 192
7.12 Acknowledgements . . . . . 195
7.13 References . . . . . 195