

STATISTICAL MECHANICS OF PERIODIC
FRUSTRATED ISING SYSTEMS

Rainer Liebmann
Max-Planck-Institut für Festkörperforschung *)
Heisenbergstr. 1, D-7000 Stuttgart 80

CONTENTS

1. Introduction and survey	1
1.1 Critical phenomena at second order phase transitions	4
1.2 Scope of this book	6
2. One-dimensional frustrated Ising systems	7
2.1 Periodic ANNNI-chain	7
2.1.1 Groundstate degeneracy of the ANNNI-chain	9
2.1.2 Periodic ANNNI-chain for $T \neq 0$	11
2.2 Decorated chains	15
2.3 Partially frustrated chains	20
2.3.1 Periodic frustrated chains	21
2.3.2 Random frustrated chain	22
3. Two-dimensional frustrated Ising systems	26
3.1 Transformations of Ising systems	26
3.1.1 Duality transformation	29
3.1.2 Decimation transformation	34
a) Decoration-iteration transformation	34
b) Star-triangle transformation	35
3.1.3 Connection between different lattices	37

*)

present address: AEG Aktiengesellschaft, Sedanstr. 10, D-7900 Ulm/FRG

3.2	Triangular lattice	38
3.2.1	Estimation of the groundstate degeneracy	39
	a) Simple lower bound	41
	b) Pauling method	42
	c) Systematic cluster approximation	42
3.2.2	Partition function and exact GS entropy of the isotropic system	46
3.2.3	Specific heat near the frustration points ($J_1 = J_2$)	48
3.2.4	Pair correlation function, disorder lines ($J_1 = J_2$)	51
3.2.5	Mapping to the quantum xy-chain and to the kinetic nn Ising chain	54
3.3	Further frustrated systems with noncrossing interactions	61
3.3.1	Union-Jack lattice	61
3.3.2	Villain's odd model and its generalizations	68
	a) Groundstates and phase diagrams	68
	b) Correlation functions	70
	c) Periodical layered models	71
	d) Chessboard model	74
3.3.3	Hexagon lattice	75
3.3.4	Pentagon lattice	76
3.3.5	Kagomé lattice	77
3.3.6	Connection between groundstate degeneracy and existence of a phase transition at $T_c = 0$	81
3.4	Frustrated Ising systems with crossing interactions	82
3.4.1	2d ANNNI-model	82
3.4.2	Brick model	87
3.4.3	Frustrated triangular lattice with nnn-interactions and magnetic field	88
	a) Additional nnn-interactions J_2	89
	b) Additional magnetic field H	93
	c) Corresponding lattice gas model	97
3.4.4	Square lattice with competing nn- and nnn-interactions: relation to vertex models	99
	a) System without magnetic field	99
	b) Systems with magnetic field	101
	c) Connection to vertex models	105
4.	Three-dimensional frustrated Ising systems	109
4.1	fcc antiferromagnet	109
4.2	Fully and partially frustrated simple cubic lattice	112

4.3	AF pyrochlore model	117
4.4	ANNNI-model	122
4.5	fcc four-spin (quartet) model	127
5.	Conclusion	131
	References	133