

# Contents

|                                                         |             |
|---------------------------------------------------------|-------------|
| <b>Introduction</b>                                     | <b>XIII</b> |
| <b>Notation and Terminology</b>                         | <b>XVII</b> |
| <b>Chapter 1. Hochschild Homology</b>                   | <b>1</b>    |
| 1.0 Chain Complexes                                     | 2           |
| 1.1 Hochschild Homology                                 | 8           |
| 1.2 The Trace Map and Morita Invariance                 | 16          |
| 1.3 Derivations, Differential Forms                     | 23          |
| 1.4 Nonunital Algebras and Excision                     | 28          |
| 1.5 Hochschild Cohomology, Cotrace, Duality             | 37          |
| 1.6 Simplicial Modules                                  | 44          |
| Bibliographical Comments on Chapter 1                   | 48          |
| <b>Chapter 2. Cyclic Homology of Algebras</b>           | <b>50</b>   |
| 2.1 Definition of Cyclic Homology                       | 52          |
| 2.2 Connes' Exact Sequence, Morita Invariance, Excision | 60          |
| 2.3 Differential Forms, de Rham Cohomology              | 67          |
| 2.4 Cyclic Cohomology                                   | 71          |
| 2.5 Cyclic Modules                                      | 74          |
| 2.6 Non-commutative Differential Forms                  | 82          |
| Bibliographical Comments on Chapter 2                   | 87          |
| <b>Chapter 3. Smooth Algebras and Other Examples</b>    | <b>88</b>   |
| 3.1 Tensor Algebra                                      | 89          |
| 3.2 Symmetric Algebras                                  | 93          |
| 3.3 Universal Enveloping Algebras of Lie Algebras       | 96          |
| 3.4 Smooth Algebras                                     | 100         |
| 3.5 André-Quillen Homology                              | 106         |
| 3.6 Deligne Cohomology                                  | 111         |
| Bibliographical Comments on Chapter 3                   | 113         |

|                                                                           |            |
|---------------------------------------------------------------------------|------------|
| <b>Chapter 4. Operations on Hochschild and Cyclic Homology</b>            | <b>114</b> |
| 4.1 Conjugation and Derivation                                            | 115        |
| 4.2 Shuffle Product in Hochschild Homology                                | 122        |
| 4.3 Cyclic Shuffles and Künneth Sequence for $HC$                         | 126        |
| 4.4 Product, Coproduct in Cyclic Homology                                 | 132        |
| 4.5 $\lambda$ -Decomposition for Hochschild Homology                      | 138        |
| 4.6 $\lambda$ -Decomposition for Cyclic Homology                          | 148        |
| Bibliographical Comments on Chapter 4                                     | 154        |
| <b>Chapter 5. Variations on Cyclic Homology</b>                           | <b>155</b> |
| 5.1 The Periodic and Negative Theories                                    | 156        |
| 5.2 Dihedral and Quaternionic Homology                                    | 165        |
| 5.3 Differential Graded Algebras                                          | 173        |
| 5.4 Commutative Graded Differential Graded Algebras                       | 180        |
| 5.5 Bivariant Cyclic Cohomology                                           | 188        |
| 5.6 Topological Algebras, Entire Cyclic Cohomology                        | 193        |
| Bibliographical Comments on Chapter 5                                     | 197        |
| <b>Chapter 6. The Cyclic Category, Tor and Ext Interpretation</b>         | <b>198</b> |
| 6.1 Connes Cyclic Category $\Delta C$ and the Category $\Delta S$         | 199        |
| 6.2 Tor and Ext Interpretation of $HH$ and $HC$                           | 208        |
| 6.3 Crossed Simplicial Groups                                             | 211        |
| 6.4 The Category of Finite Sets and the $\lambda$ -Decomposition          | 217        |
| Bibliographical Comments on Chapter 6                                     | 222        |
| <b>Chapter 7. Cyclic Spaces and <math>S^1</math>-Equivariant Homology</b> | <b>223</b> |
| 7.1 Cyclic Sets and Cyclic Spaces                                         | 224        |
| 7.2 Cyclic Homology and $S^1$ -Equivariant Homology                       | 230        |
| 7.3 Examples of Cyclic Sets and the Free Loop Space                       | 237        |
| 7.4 Hochschild Homology and Cyclic Homology of Group Algebras             | 246        |
| Bibliographical Comments on Chapter 7                                     | 252        |
| <b>Chapter 8. Chern Character</b>                                         | <b>253</b> |
| 8.1 The Classical Chern Character à la Chern-Weil                         | 254        |
| 8.2 The Grothendieck Group $K_0$                                          | 257        |
| 8.3 The Chern Character from $K_0$ to Cyclic Homology                     | 260        |
| 8.4 The Dennis Trace Map and the Generalized Chern Character              | 265        |
| 8.5 The Bass Trace Conjecture and the Idempotent Conjecture               | 273        |
| Bibliographical Comments on Chapter 8                                     | 275        |

|                                                                      |     |
|----------------------------------------------------------------------|-----|
| <b>Chapter 9. Classical Invariant Theory</b> .....                   | 277 |
| 9.1 The Fundamental Theorems of Invariant Theory .....               | 278 |
| 9.2 Coinvariant Theory and the Trace Map .....                       | 280 |
| 9.3 Cayley-Hamilton and Amitsur-Levitzki Formulas .....              | 283 |
| 9.4 Proofs of the Fundamental Theorems .....                         | 287 |
| 9.5 Invariant Theory for the Orthogonal and Symplectic Groups ..     | 289 |
| Bibliographical Comments on Chapter 9 .....                          | 293 |
| <b>Chapter 10. Homology of Lie Algebras of Matrices</b> .....        | 295 |
| 10.1 Homology of Lie Algebras .....                                  | 297 |
| 10.2 Homology of the Lie Algebra $gl(A)$ .....                       | 302 |
| 10.3 Stability and First Obstruction to Stability .....              | 311 |
| 10.4 Homology with Coefficients in the Adjoint Representation ....   | 316 |
| 10.5 The Symplectic and Orthogonal Cases .....                       | 319 |
| 10.6 Non-commutative Homology of the Lie Algebra of Matrices ...     | 322 |
| Bibliographical Comments on Chapter 10 .....                         | 336 |
| <b>Chapter 11. Algebraic <math>K</math>-Theory</b> .....             | 337 |
| 11.1 The Bass-Whitehead Group $K_1$ and the Milnor Group $K_2$ ....  | 338 |
| 11.2 Higher Algebraic $K$ -Theory .....                              | 345 |
| 11.3 Algebraic $K$ -Theory and Cyclic Homology of Nilpotent Ideals . | 357 |
| 11.4 Absolute and Relative Chern Characters .....                    | 367 |
| 11.5 Secondary Characteristic Classes .....                          | 371 |
| Bibliographical Comments on Chapter 11 .....                         | 376 |
| <b>Chapter 12. Non-commutative Differential Geometry</b> .....       | 377 |
| 12.1 Foliations and the Godbillon-Vey Invariant .....                | 378 |
| 12.2 Fredholm Modules and Index Theorems .....                       | 381 |
| 12.3 Novikov Conjecture on Higher Signatures .....                   | 386 |
| 12.4 The $K$ -Theoretic Analogue of the Novikov Conjecture .....     | 391 |
| <b>Appendices</b>                                                    |     |
| A. Hopf Algebras .....                                               | 395 |
| B. Simplicial .....                                                  | 401 |
| C. Homology of Discrete Groups and Small Categories .....            | 409 |
| D. Spectral Sequences .....                                          | 416 |
| E. Smooth Algebras (by María O. Ronco) .....                         | 420 |
| <b>References</b> .....                                              | 431 |
| <b>Symbols</b> .....                                                 | 445 |
| <b>Index</b> .....                                                   | 449 |