## **Contents**

	Preface
Chapter 1	Background
1	Cobordism Theory
2	Characteristic classes
3	Pontrjagin classes of quaternionic projective spaces 5
4	Characteristic classes and invariants
5	Representations and vector bundles
6	Multiplicative sequences and genera
7	Calculation of $\varphi(P_k(\mathbb{H}))$
8	Complex genera
Chapter 2	Elliptic genera
- 1	The Weierstraß p-function
2	Construction of elliptic genera
3	An excursion on the lemniscate
4	Geometric complement on the addition theorem32
Chapter 3	A universal addition theorem for genera
<b>-</b> 1	Virtual submanifolds
2	A universal genus
Chapter 4	Multiplicativity in fibre bundles
<b>-</b> 1	41
2	<b>.</b>
3	
4	46
4	
,	Multiplicativity and strict multiplicativity



Chapter 5	The Atiyah-Singer index theorem
1	Elliptic operators and elliptic complexes
2	The index of an elliptic complex
3	The de Rham complex
4	The Dolbeault complex
5	The signature as an index
6	The equivariant index
7	The equivariant $\chi_y$ -genus for $S^1$ -actions
8	The equivariant signature for $S^1$ -actions
Chapter 6	Twisted operators and genera
1	Motivation for elliptic genera after Ed Witten73
2	The expansion at the cusp 0
3	The Witten genus
4	The Witten genus and the Lie group $E_8$
5	Plumbing of manifolds
Chapter 7	Riemann-Roch and elliptic genera in the complex case
1	Elliptic genera of level $N$
2	The values at the cusps
3	The equivariant case and multiplicativity 100
4	The loop space and the expansion at a cusp 102
5	The differential equation
6	The modular curve
Chapter 8	A divisibility theorem for elliptic genera
1	The theorem of Ochanine
2	Proof of Ochanine's theorem

Contents

Appendix I	Modular forms
1	Fundamental concepts
2	Examples of modular forms
3	The Weierstraß $\wp$ -function as a Jacobi form 128
4	Some special functions and modular forms
5	Theta functions, divisors, and elliptic functions
6	Elliptic genera of level $N \dots 149$
7	$N$ -division points and $\Gamma_1(N) \backslash \mathfrak{h} \ldots 153$
Appendix II	The Dirac operator
1	The solution
2	The problem
Appendix III	Elliptic genera of level $N$ for complex manifolds
Appendix IV	Zolotarev polynomials and the modular curve $X_1(N)$
1	Zolotarev polynomials
2	Interpretation as an algebraic curve
3	The differential equation — revisited
4	Modular interpretation of Zolotarev polynomials 194
5	The embedding of the modular curve
6	Applications to elliptic genera
	Bibliography
	Index
	<b>Symbols</b>