

# Contents

<b>1. Introduction</b> .....	<b>1</b>
------------------------------	----------

---

## Part I Methods

---

<b>2. Reductive Perturbation Method</b> .....	<b>5</b>
2.1 Oscillators Versus Fields of Oscillators .....	5
2.2 The Stuart-Landau Equation .....	8
2.3 Onset of Oscillations in Distributed Systems .....	13
2.4 The Ginzburg-Landau Equation .....	17
<b>3. Method of Phase Description I</b> .....	<b>22</b>
3.1 Systems of Weakly Coupled Oscillators .....	22
3.2 One-Oscillator Problem .....	24
3.3 Nonlinear Phase Diffusion Equation .....	28
3.4 Representation by the Floquet Eigenvectors .....	29
3.5 Case of the Ginzburg-Landau Equation .....	32
<b>4. Method of Phase Description II</b> .....	<b>35</b>
4.1 Systematic Perturbation Expansion .....	35
4.2 Generalization of the Nonlinear Phase Diffusion Equation .....	41
4.3 Dynamics of Slowly Varying Wavefronts .....	46
4.4 Dynamics of Slowly Phase-Modulated Periodic Waves .....	54

---

## Part II Applications

---

<b>5. Mutual Entrainment</b> .....	<b>60</b>
5.1 Synchronization as a Mode of Self-Organization .....	60
5.2 Phase Description of Entrainment .....	62
5.2.1 One Oscillator Subject to Periodic Force ..	62
5.2.2 A Pair of Oscillators with Different Frequencies .....	65
5.2.3 Many Oscillators with Frequency Distribution .....	66
5.3 Calculation of $I$ for a Simple Model .....	67
5.4 Soluble Many-Oscillator Model Showing Synchronization- Desynchronization Transitions .....	68

5.5 Oscillators Subject to Fluctuating Forces .....	78
5.5.1 One Oscillator Subject to Stochastic Forces .....	78
5.5.2 A Pair of Oscillators Subject to Stochastic Forces .....	80
5.5.3 Many Oscillators Which are Statistically Identical .....	82
5.6 Statistical Model Showing Synchronization-Desynchronization Transitions .....	82
5.7 Bifurcation of Collective Oscillations .....	84
<b>6. Chemical Waves</b> .....	89
6.1 Synchronization in Distributed Systems .....	89
6.2 Some Properties of the Nonlinear Phase Diffusion Equation .....	91
6.3 Development of a Single Target Pattern .....	93
6.4 Development of Multiple Target Patterns .....	101
6.5 Phase Singularity and Breakdown of the Phase Description .....	103
6.6 Rotating Wave Solution of the Ginzburg-Landau Equation .....	106
<b>7. Chemical Turbulence</b> .....	111
7.1 Universal Diffusion-Induced Turbulence .....	111
7.2 Phase Turbulence Equation .....	114
7.3 Wavefront Instability .....	120
7.4 Phase Turbulence .....	127
7.5 Amplitude Turbulence .....	132
7.6 Turbulence Caused by Phase Singularities .....	137
<b>Appendix</b> .....	141
A. Plane Wave Solutions of the Ginzburg-Landau Equation .....	141
B. The Hopf Bifurcation for the Brusselator .....	144
<b>References</b> .....	149
<b>Subject Index</b> .....	155