

Contents

1	Introduction	1
1.1	The Hybrid Displacement Boundary Element Model	1
1.2	Historical Development of Variational Principles	3
1.3	Variational Principles and Finite Element Models	6
1.4	Boundary Element Method Fundamentals	10
1.5	Boundary Element Variational Formulations	13
2	Potential Problems	16
2.1	Introduction	16
2.2	Indicial Notation	18
2.3	Basic Equations	20
2.4	Generalized Variational Principle	23
2.5	Derivation of the Model	28
2.5.1	Definition of Fundamental Solution	29
2.5.2	Approximation for the Domain Variable	30

2.5.3	Approximations for the Boundary Variables	32
2.5.4	Final System of Equations	33
2.5.5	Solution on the Boundary	39
2.5.6	Solution at Internal Points	39
2.6	Symmetry of the Stiffness Matrix	40
3	Numerical Aspects in Potential Problems	44
3.1	Introduction	44
3.2	The Constant Element	46
3.2.1	Matrix F for Constant Elements	48
3.2.2	Matrix G for Constant Elements	57
3.2.3	Matrix L for Constant Elements	59
3.2.4	Equivalent Nodal Fluxes	60
3.3	The Quadratic Element	60
3.3.1	Matrix F for Quadratic Elements	64
3.3.2	Matrix G for Quadratic Elements	72
3.3.3	Matrix L for Quadratic Elements	74
3.3.4	Equivalent Nodal Fluxes	74
3.4	The Vector B	76

4	Elastostatics	82
4.1	Introduction	82
4.2	Basic Relations in Linear Elastostatics	83
4.3	Modified Variational Principle	86
4.4	Derivation of the Model	92
4.4.1	Fundamental Solution	92
4.4.2	Approximation for the Domain Variable	94
4.4.3	Approximation for Boundary Variables	97
4.4.4	Final System of Equations	98
4.4.5	Solution on the Boundary	103
4.4.6	Solution at Internal Points	103
4.4.7	Symmetry of the Stiffness Matrix	104
5	Numerical Aspects in Elastostatics Problems	107
5.1	Introduction	107
5.2	The Constant Element	110
5.2.1	Matrix F for Constant Elements	111
5.2.2	Matrix G for Constant Elements	121
5.2.3	Matrix L for Constant Elements	123
5.2.4	Load Vector	124
5.3	The Quadratic Element	124

5.3.1	Matrix $\mathbf{F}$ for Quadratic Elements	127
5.3.2	Matrix $\mathbf{G}$ for Quadratic Elements	137
5.3.3	Matrix $\mathbf{L}$ for Quadratic Elements	139
5.3.4	Load Vector	139
5.4	Computation of the Submatrices $\mathcal{F}^{ii}$	141
5.5	Body Forces	143
5.5.1	Transformation of the Domain Integrals into Boundary Integrals	145
6	Numerical Applications	149
6.1	Introduction	149
6.2	Examples for Potential Problems	150
6.2.1	Constant Elements	150
6.2.2	Quadratic Elements	158
6.3	Elasticity Problems	167
6.3.1	Constant Elements	168
6.3.2	Quadratic Elements	175
7	Conclusions	181
8	Bibliography	186