Contents

Introduction: Long-Time Behavior of Evolution Inclusion Solutions in Earth Data Analysis xiii
References xxxi

Part I Longtime Behavior of Autonomous Differential-Operator Systems Solutions for Earth Data Processing

1 Abstract Theory of Multivalued Semiflows 3
1.1 ω-Limit Sets and Global Attractors of Multivalued Semiflows 5
1.2 Comparison Between Trajectory and Global Attractors for Evolution Systems 17
1.2.1 The Main Definitions 18
1.2.2 Main Results 21
1.2.3 Some Model Applications 30
References 33

2 Auxiliary Properties of Evolution Inclusions Solutions for Earth Data Processing 37
2.1 Preliminaries 38
2.2 Pointwise Pseudomonotone Maps 43
2.3 Auxiliary Properties of Solutions for the First Order Evolution 53
2.3.1 The Setting of the Problem 53
2.3.2 Preliminaries 54
2.3.3 Supplementary Properties of Solutions 56
2.4 Asymptotic Behavior of the First Order Evolution Inclusions 61
2.4.1 Existence of the Global Attractor 62
2.4.2 Existence of the Trajectory Attractor 63
2.4.3 Comments 66
2.4.4 Conclusion 67

2.5 Auxiliary Properties of Solutions for the Second Order Evolution Inclusions 67
2.5.1 Preliminary Results 69
2.5.2 Auxiliary Properties of the Resolving Operator 71
2.6 Auxiliary Properties of Solutions for the Second Order Evolution Inclusions 79
2.7 Asymptotic Behavior of the Second-Order Evolution Inclusions 86
2.7.1 Existence of the Global Attractor 87
2.7.2 Existence of the Trajectory Attractor 93
2.7.3 Auxiliary Properties of the Global and Trajectory Attractors 96
2.8 Applications 100
2.8.1 Climate Energy Balance Model 100
2.8.2 Application for General Classes High-Order Nonlinear PDEs 101
2.8.3 Application for Chemotaxis Processes 102
2.8.4 Applications for Damped Viscoelastic Fields with Short Memory 105
2.8.5 Applications for Nonsmooth Autonomous Piezoelectric Fields 110
References 116

3 Attractors for Lattice Dynamical Systems 119
3.1 Existence of Solutions 120
3.2 A Priori Estimates 141
3.2.1 Existence of a Bounded Absorbing Set 141
3.2.2 Estimate of the Tails 142
3.3 Existence of the Global Attractor 144
3.4 Approximation of the Attractor 151
3.5 Application for Discrete Climate Energy Balance Model 158
References 159

Part II Longtime Behavior of Nonautonomous Differential-Operator Systems Solutions for Earth Data Processing

4 On Global Attractors of Multivalued Semiprocesses and Nonautonomous Evolution Inclusions 163
4.1 ω-Limit Sets and Global Attractors of Multivalued Semiprocesses 164
4.2 Global Attractors for Nonautonomous Differential Inclusions 177
4.2.1 Abstract Setting: Construction of the Multivalued Semiprocess 177

4.2.2 Global Attractors of Nonautonomous Reaction-Diffusion Inclusions 182
4.3 Applications for Chemical Kinetics Processes and Fields 197
References 197

5 On the Kneser's Property for the Complex Ginzburg–Landau Equation and the Lotka–Volterra System with Diffusion 199
5.1 Setting of the Problem 202
5.2 The Kneser's Property for Reaction-Diffusion Systems 204
5.2.1 Application to the Complex Ginzburg–Landau Equation 220
5.2.2 Application to the Lotka–Volterra System with Diffusion 221
5.3 Connectedness of Attractors for Reaction-Diffusion Systems 222
5.3.1 Application to the Complex Ginzburg–Landau Equation 227
5.3.2 Application to the Lotka–Volterra System with Diffusion 228
References 229

6 Pullback Attractors for a Class of Extremal Solutions of the 3D Navier–Stokes System 231
6.1 Pullback Attractors for Multivalued Processes 232
6.2 Setting of the Problem and Main Results 235
6.3 Relationship with the Attractor of the 3D Navier–Stokes System 250
6.4 Applications for Hydrodynamic Problems 253
References 256

7 Properties of Resolving Operator for Nonautonomous Evolution Inclusions: 259
7.1 New Theorems for Existence Solutions for Skrypnik's Type Operators 261
7.1.1 Problem Definition 262
7.1.2 The Class $\mathcal{H}(X^*)$ 264
7.1.3 Classes of MultiValued Maps 264
7.1.4 The Main Results 267
7.2 Noncoercive Evolution Inclusions for S_k Type Operators 276
7.2.1 Preliminaries: On Some Classes of Multivalued Maps 279
7.2.2 Setting of the Problem 281
7.2.3 Main Results 285
7.2.4 Applications 298
7.3 Functional-Topological Properties of the Resolving Operator for the Evolution Inclusion 300
7.3.1 The Setting of the Problem 301
7.3.2 Main Results 301

7.4 Auxiliary Properties of Solutions for the Nonautonomous First-Order Evolution Inclusions with Uniformly Coercive Mappings, Long-Time Behavior, and Pullback Attractors 313
7.5 Applications 317
References 317

A Functional Spaces: The Embedding and Approximation Theorems 321
References 328

Index 329