

Contents

Preface	vii
I Skeletons and dessins	
1 Graphs	3
1.1 Graphs and trees	3
1.1.1 Graphs	3
1.1.2 Trees	6
1.1.3 Dynkin diagrams	7
1.2 Skeletons	9
1.2.1 Ribbon graphs	9
1.2.2 Regions	12
1.2.3 The fundamental group	16
1.2.4 First applications	22
1.3 Pseudo-trees	26
1.3.1 Admissible trees	26
1.3.2 The counts	31
1.3.3 The associated lattice	36
2 The groups Γ and $\mathbb{B}_3$	41
2.1 The modular group $\Gamma := PSL(2, \mathbb{Z})$	41
2.1.1 The presentation of Γ	41
2.1.2 Subgroups	47
2.2 The braid group $\mathbb{B}_3$	50
2.2.1 Artin's braid groups $\mathbb{B}_n$	50
2.2.2 The Burau representation	54
2.2.3 The group $\mathbb{B}_3$	57
3 Trigonal curves and elliptic surfaces	63
3.1 Trigonal curves	63
3.1.1 Basic definitions and properties	63
3.1.2 Singular fibers	71
3.1.3 Special geometric structures	76

3.2	Elliptic surfaces	79
3.2.1	The local theory	79
3.2.2	Compact elliptic surfaces	83
3.3	Real structures	90
3.3.1	Real varieties	91
3.3.2	Real trigonal curves and real elliptic surfaces	96
3.3.3	Lefschetz fibrations	101
4	Dessins	109
4.1	Dessins	109
4.1.1	Trichotomic graphs	109
4.1.2	Deformations	115
4.2	Trigonal curves <i>via</i> dessins	118
4.2.1	The correspondence theorems	118
4.2.2	Complex curves	120
4.2.3	Generic real curves	131
4.3	First applications	137
4.3.1	Ribbon curves	137
4.3.2	Elliptic Lefschetz fibrations revisited	142
5	The braid monodromy	146
5.1	The Zariski–van Kampen theorem	146
5.1.1	The monodromy of a proper n -gonal curve	146
5.1.2	The fundamental groups	152
5.1.3	Improper curves: slopes	158
5.2	The case of trigonal curves	164
5.2.1	Monodromy <i>via</i> skeletons	164
5.2.2	Slopes	170
5.2.3	The strategy	173
5.3	Universal curves	177
5.3.1	Universal curves	177
5.3.2	The irreducibility criteria	179
II	Applications	
6	The metabelian invariants	183
6.1	Dihedral quotients	183
6.1.1	Uniform dihedral quotients	183
6.1.2	Geometric implications	187

6.2	The Alexander module	190
6.2.1	Statements	190
6.2.2	Proof of Theorem 6.16: the case $N \geq 7$	193
6.2.3	Congruence subgroups (the case $N \leq 5$)	196
6.2.4	The parabolic case $N = 6$	199
7	A few simple computations	203
7.1	Trigonal curves in Σ_2	203
7.1.1	Proper curves in Σ_2	203
7.1.2	Perturbations of simple singularities	207
7.2	Sextics with a non-simple triple point	213
7.2.1	A gentle introduction to plane sextics	213
7.2.2	Classification and fundamental groups	220
7.2.3	A summary of further results	221
7.3	Plane quintics	224
8	Fundamental groups of plane sextics	227
8.1	Statements	227
8.1.1	Principal results	227
8.1.2	Beginning of the proof	228
8.2	A distinguished point of type E	231
8.2.1	A point of type E_8	232
8.2.2	A point of type E_7	238
8.2.3	A point of type E_6	244
8.3	A distinguished point of type D	259
8.3.1	A point of type D_p , $p \geq 6$	259
8.3.2	A point of type D_5	263
8.3.3	A point of type D_4	269
9	The transcendental lattice	275
9.1	Extremal elliptic surfaces without exceptional fibers	275
9.1.1	The tripod calculus	275
9.1.2	Proofs and further observations	277
9.2	Generalizations and examples	281
9.2.1	A computation <i>via</i> the homological invariant	281
9.2.2	An example	284
10	Monodromy factorizations	288
10.1	Hurwitz equivalence	288
10.1.1	Statement of the problem	288
10.1.2	$\mathbb{F}_n$ -valued factorizations	291
10.1.3	$\mathbb{S}_n$ -valued factorizations	292

10.2 Factorizations in Γ	297
10.2.1 Exponential examples	297
10.2.2 2-factorizations	301
10.2.3 The transcendental lattice	307
10.2.4 2-factorizations <i>via</i> matrices	313
10.3 Geometric applications	316
10.3.1 Extremal elliptic surfaces	316
10.3.2 Ribbon curves <i>via</i> skeletons	318
10.3.3 Maximal Lefschetz fibrations are algebraic	323

Appendices

A An algebraic complement	329
A.1 Integral lattices	329
A.1.1 Nikulin's theory of discriminant forms	329
A.1.2 Definite lattices	331
A.2 Quotient groups	335
A.2.1 Zariski quotients	335
A.2.2 Auxiliary lemmas	336
A.2.3 Alexander module and dihedral quotients	337
B Bigonal curves in Σ_d	340
B.1 Bigonal curves in Σ_d	340
B.2 Plane quartics, quintics, and sextics	344
C Computer implementations	346
C.1 GAP implementations	346
C.1.1 Manipulating skeletons in GAP	346
C.1.2 Proof of Theorem 6.16	352
D Definitions and notation	359
D.1 Common notation	359
D.1.1 Groups and group actions	359
D.1.2 Topology and homotopy theory	360
D.1.3 Algebraic geometry	362
D.1.4 Miscellaneous notation	364
D.2 Index of notation	365
Bibliography	369
Index of figures	379
Index of tables	382
Index	383