

Contents

1. Goal	
1.1	Order and Disorder: Some Typical Phenomena 1
1.2	Some Typical Problems and Difficulties 12
1.3	How We Shall Proceed 15
2. Probability	
2.1	Object of Our Investigations: The Sample Space 17
2.2	Random Variables 19
2.3	Probability 20
2.4	Distribution 21
2.5	Random Variables with Densities 24
2.6	Joint Probability 26
2.7	Mathematical Expectation $E(X)$, and Moments 28
2.8	Conditional Probabilities 29
2.9	Independent and Dependent Random Variables 30
2.10*	Generating Functions and Characteristic Functions 31
2.11	A Special Probability Distribution: Binomial Distribution 33
2.12	The Poisson Distribution 36
2.13	The Normal Distribution (Gaussian Distribution) 37
2.14	Stirling's Formula 39
2.15*	Central Limit Theorem 39
3. Information	
3.1	Some Basic Ideas 41
3.2*	Information Gain: An Illustrative Derivation 46
3.3	Information Entropy and Constraints 48
3.4	An Example from Physics: Thermodynamics 53
3.5*	An Approach to Irreversible Thermodynamics 57
3.6	Entropy—Curse of Statistical Mechanics? 66
4. Chance	
4.1	A Model of Brownian Movement 69
4.2	The Random Walk Model and Its Master Equation 75
4.3*	Joint Probability and Paths. Markov Processes. The Chapman-Kolmogorov Equation. Path Integrals 79

* Sections with an asterisk in the heading may be omitted during a first reading.

4.4*	How to Use Joint Probabilities. Moments. Characteristic Function. Gaussian Processes	85
4.5	The Master Equation	88
4.6	Exact Stationary Solution of the Master Equation for Systems in Detailed Balance	89
4.7*	The Master Equation with Detailed Balance. Symmetrization, Eigenvalues and Eigenstates	92
4.8*	Kirchhoff's Method of Solution of the Master Equation	95
4.9*	Theorems about Solutions of the Master Equation	97
4.10	The Meaning of Random Processes, Stationary State, Fluctuations, Recurrence Time	98
4.11*	Master Equation and Limitations of Irreversible Thermodynamics	102
 5. Necessity		
5.1	Dynamic Processes	105
5.2*	Critical Points and Trajectories in a Phase Plane. Once Again Limit Cycles	113
5.3*	Stability	120
5.4	Examples and Exercises on Bifurcation and Stability	126
5.5*	Classification of Static Instabilities, or an Elementary Approach to Thom's Theory of Catastrophes	133
 6. Chance and Necessity		
6.1	Langevin Equations: An Example	147
6.2*	Reservoirs and Random Forces	152
6.3	The Fokker-Planck Equation	158
6.4	Some Properties and Stationary Solutions of the Fokker-Planck-Equation	165
6.6	Time-Dependent Solutions of the Fokker-Planck Equation	172
6.6*	Solution of the Fokker-Planck Equation by Path Integrals	176
6.7	Phase Transition Analogy	179
6.8	Phase Transition Analogy in Continuous Media: Space-Dependent Order Parameter	186
 7. Self-Organization		
7.1	Organization	191
7.2	Self-Organization	194
7.3	The Role of Fluctuations: Reliability or Adaptibility? Switching	200
7.4*	Adiabatic Elimination of Fast Relaxing Variables from the Fokker-Planck Equation	202
7.5*	Adiabatic Elimination of Fast Relaxing Variables from the Master Equation	204
7.6	Self-Organization in Continuously Extended Media. An Outline of the Mathematical Approach	205

7.7*	Generalized Ginzburg-Landau Equations for Nonequilibrium Phase Transitions	206
7.8*	Higher-Order Contributions to Generalized Ginzburg-Landau Equations	216
7.9*	Scaling Theory of Continuously Extended Nonequilibrium Systems	219
7.10*	Soft-Mode Instability	222
7.11*	Hard-Mode Instability	226
 8. Physical Systems		
8.1	Cooperative Effects in the Laser: Self-Organization and Phase Transition	229
8.2	The Laser Equations in the Mode Picture	230
8.3	The Order Parameter Concept	231
8.4	The Single-Mode Laser	232
8.5	The Multimode Laser	235
8.6	Laser with Continuously Many Modes. Analogy with Superconductivity	237
8.7	First-Order Phase Transitions of the Single-Mode Laser	240
8.8	Hierarchy of Laser Instabilities and Ultrashort Laser Pulses	243
8.9	Instabilities in Fluid Dynamics: The Bénard and Taylor Problems	249
8.10	The Basic Equations	250
8.11	The Introduction of New Variables	252
8.12	Damped and Neutral Solutions ($R \leq R_c$)	254
8.13	Solution Near $R = R_c$ (Nonlinear Domain). Effective Langevin Equations	258
8.14	The Fokker-Planck Equation and Its Stationary Solution	262
8.15	A Model for the Statistical Dynamics of the Gunn Instability Near Threshold	266
8.16	Elastic Stability: Outline of Some Basic Ideas	270
 9. Chemical and Biochemical Systems		
9.1	Chemical and Biochemical Reactions	275
9.2	Deterministic Processes, Without Diffusion, One Variable	275
9.3	Reaction and Diffusion Equations	280
9.4	Reaction-Diffusion Model with Two or Three Variables: The Brusselator and the Oregonator	282
9.5	Stochastic Model for a Chemical Reaction Without Diffusion. Birth and Death Processes. One Variable	289
9.6	Stochastic Model for a Chemical Reaction with Diffusion. One Variable	294
9.7*	Stochastic Treatment of the Brusselator Close to Its Soft-Mode Instability	298
9.8	Chemical Networks	302

10. Applications to Biology

10.1 Ecology, Population-Dynamics	305
10.2 Stochastic Models for a Predator-Prey System	309
10.3 A Simple Mathematical Model for Evolutionary Processes	310
10.4 A Model for Morphogenesis	311
10.5 Order Parameters and Morphogenesis	314
10.6 Some Comments on Models of Morphogenesis	325

11. Sociology and Economics

11.1 A Stochastic Model for the Formation of Public Opinion	327
11.2 Phase Transitions in Economics	329

12. Chaos

12.1 What is Chaos?	333
12.2 The Lorenz Model. Motivation and Realization	334
12.3 How Chaos Occurs	336
12.4 Chaos and the Failure of the Slaving Principle	341
12.5 Correlation Function and Frequency Distribution	343
12.6 Discrete Maps, Period Doubling, Chaos, Intermittency	345

13. Some Historical Remarks and Outlook 351

References, Further Reading, and Comments 355

Subject Index 367