

Contents

List of Symbols	xxi
-----------------------	-----

Part I Fundamental Laws

1 Preliminaries	3
1.1 Flow Kinematics	3
1.1.1 Lagrangian and Eulerian Representation	3
1.1.2 Deformation and Rotation	6
1.2 Thermodynamics of Sea Water	10
1.2.1 Salt Concentration and Salinity	11
1.2.2 Additive State Variables	12
1.2.3 First Law of Thermodynamics	14
1.2.4 Second Law of Thermodynamics	16
1.2.5 Thermodynamic Potentials	17
1.2.6 Equation of State	21
1.2.7 Specific Heat	23
2 Conservation Laws for Moving Fluids	25
2.1 General Form of Conservation Equations	25
2.2 Mass Conservation	28
2.2.1 Total Mass and Salt Conservation Equation	29
2.2.2 Boundary Conditions for the Fluxes of Total Mass and Salt	30
2.3 Conservation of Momentum	31
2.3.1 Stresses, Pressure and Frictional Forces	32
2.3.2 Boundary Condition for the Momentum Flux	38
2.3.3 Conservation Equations on the Rotating Earth	39
2.3.4 The Force of Gravity on the Earth	42

2.4	Energy Conservation	47
2.4.1	Contributions to the Change of Energy in a Material Volume	48
2.4.2	Mechanical Energy	49
2.4.3	Internal Energy and Enthalpy	50
2.4.4	Total Energy and Total Enthalpy	51
2.4.5	Boundary Condition for the Enthalpy Flux	53
2.5	Entropy Budget	54
2.5.1	Entropy Sources and Flux-Gradient Relations	54
2.5.2	Onsager Relations	56
2.6	Temperature Equations	57
2.6.1	In-situ Temperature	58
2.6.2	Conservative Temperature	58
2.6.3	Potential Temperature	61
2.6.4	Conservative Temperature as a State Variable	63
2.7	Density Variables	65
2.7.1	Potential Density	65
2.7.2	Neutral Surface Elements	66
2.8	Molecular and Turbulent Transports	68
2.8.1	Magnitude of Molecular Transports	68
2.8.2	Reynolds and Hesselberg Averaging	70
2.9	The State of Rest	72
2.9.1	Hydrostatic Balance	73
2.9.2	Static Stability	75
2.10	* Some Differences to Atmospheric Thermodynamics	76
2.10.1	Differences in Thermodynamics	77
2.10.2	Differences in Conservation Laws	80
2.11	Vorticity	83
2.11.1	Kinematical Properties	85
2.11.2	Dynamical Properties	87
2.11.3	Ertel's Potential Vorticity	92
2.12	* Lagrangian Concepts in Fluid Mechanics	94
2.12.1	Incompressible Fluid	97
2.12.2	Compressible Isentropic Fluid	99
2.12.3	Rotating Fluid with Gravity	100
2.12.4	Rotating Stratified Fluid	100
2.12.5	A Variational Principle for Eulerian Coordinates	101

Part II Common Approximations

3	Approximations Derived from Mode Filtering	107
3.1	A Prognostic Equation for the Pressure	107
3.2	Linear Waves	108

3.3 Filtering of Modes 112

**4 Approximations Relating to Density Changes
and Geometric Conditions 115**

4.1 Approximations Involving Density 115

 4.1.1 Anelastic Approximation 116

 4.1.2 Boussinesq Approximation 118

 4.1.3 Dynamical Role of Sea Water Compressibility 119

 4.1.4 Energetics in the Boussinesq Approximation 120

 4.1.5 Potential Vorticity in the Boussinesq Approximation..... 121

 4.1.6 Full Incompressibility and Combination
 of Salt and Heat Budgets 122

4.2 Shallow Water Approximation 123

 4.2.1 Oblate Spheroidal Coordinates 123

 4.2.2 Spherical Approximation 126

 4.2.3 Thin-Shell Approximation 128

 4.2.4 Small Aspect Ratio 129

 4.2.5 Primitive Equations 132

 4.2.6 Energetics and Potential Vorticity
 in the Shallow Water Approximation 132

5 Geostrophic and Quasi-Geostrophic Motions 135

5.1 Geostrophic Scaling 135

5.2 Quasi-Geostrophic Approximation 138

 5.2.1 Expansion for Small Parameters 140

 5.2.2 Quasi-Geostrophic Vorticity Equation 143

 5.2.3 Quasi-Geostrophic Potential Vorticity 144

 5.2.4 Boundary Conditions 145

 5.2.5 Energetics of Quasi-Geostrophic Motions 146

 5.2.6 Available Potential Energy 147

5.3 Planetary-Scale Geostrophic Motions 150

 5.3.1 The M-Representation 152

 5.3.2 Thermal Wind-Equations 152

 5.3.3 Planetary Ideal Fluid Equations 153

Part III Ocean Waves

6 Sound Waves 159

6.1 Approximations and Perturbation Expansion 159

6.2 Plane Waves 161

 6.2.1 Group Velocity I: Interference of Waves 163

 6.2.2 Energy Conservation I: Kinetic and Elastic Energy 165

 6.2.3 Sound Waves in a Mean Current 166

6.3	Propagation in a Variable Environment: WKBJ Approximation	166
6.3.1	General Wave Kinematics	168
6.3.2	Group Velocity II: Rays and Wave Packages	169
6.3.3	Energy Conservation II: Energy Flux and Group Velocity	171
6.3.4	Pathways of Sound Wave Propagation in the Ocean	173
7	Gravity Waves	177
7.1	Governing Equations	177
7.2	Plane Gravity Waves	179
7.2.1	Propagation Characteristics	179
7.2.2	Energy Conservation	182
7.3	Propagation in Variable Stratification	184
7.3.1	WKBJ Approximation for Internal Waves	185
7.3.2	Turning Points	186
7.4	The Influence of Boundaries	187
7.4.1	Reflection at a Plane Interface	187
7.4.2	Reflection at a Sloping Bottom	188
7.4.3	Vertical Modes	190
7.4.4	Accuracy of the Rigid-Lid Condition	193
7.5	Surface Waves	194
7.6	Group Velocity III: Initial Value Problems and Stationary Phase Method	196
7.7	Influence of a Mean Flow	199
7.7.1	Critical Layer Absorption	200
7.7.2	Propagation in a Geostrophic Current	201
7.7.3	Stability of Shear Flows	203
8	Long Waves	209
8.1	Long Gravity Waves	210
8.1.1	Barotropic and Baroclinic Modes	211
8.1.2	Dispersion Relation and Group Velocity	214
8.1.3	Geostrophic Adjustment	215
8.1.4	Influence of Horizontal Boundaries	219
8.1.5	Kelvin Waves	221
8.1.6	Hydraulic Control: Wave Propagation and Nonlinearity	221
8.2	Planetary Waves in Midlatitudes	224
8.2.1	Propagation Characteristics	225
8.2.2	Energy of Planetary Waves	226
8.2.3	Reflection at Meridional Boundaries	228
8.2.4	Topographic-Planetary Waves	229
8.2.5	Stationary Rossby Waves in a Baroclinic Flow over a Ridge	237
8.2.6	Spin-up of the Wind-Driven Basin Circulation	243
8.3	Equatorial Waves	248

8.3.1	Refraction due to Variations of the Coriolis Parameter	248
8.3.2	Equation for the Meridional Velocity	250
8.3.3	Meridional Eigenfunctions	251
8.3.4	Wave Solutions	254
8.3.5	Equatorial Kelvin Waves	256
8.3.6	Yanai Waves	257
8.3.7	Equatorial Rossby and Gravity Waves	258
8.3.8	Reflection at Meridional Boundaries	261
8.4	The Oceanic Waveguide	263
8.5	Influence of a Mean Flow on Planetary Waves	265
8.5.1	Modification of the Doppler Shift	266
8.5.2	Energy Transfer Between Waves and Mean Flow	268
8.5.3	Conditions for Instability	270
8.5.4	Energetics of Parcel Exchanges	280
9	* Lagrangian Theory of Ocean Waves	285
9.1	Sound Waves as Example	285
9.2	Adiabatic Invariants	286
9.3	Variational Approach to Wave Trains	288
9.4	A Rigorous Derivation	289
9.5	Rossby Waves and Internal Gravity Waves as Examples	292
9.6	Wave–Wave Interactions	296
9.6.1	Resonant Wave Triads	299
9.6.2	Interaction Theory for Random Wave Fields	300
10	Forced Waves	305
10.1	The Forcing Functions of Long Waves	306
10.2	Forced Midlatitude Waves	307
10.3	Forced Equatorial Waves	313
10.4	* Energetics of a Random Gravity Wave Field	315
10.4.1	Generation Processes	319
10.4.2	Dissipation Mechanisms	322
10.4.3	Some Prototype Balances	323
10.4.4	Resonant Transfer	324
10.4.5	The Link to Mixing	327

Part IV Oceanic Turbulence and Eddies

11	Small-Scale Turbulence	335
11.1	Kolmogorov’s Theory of Homogeneous Turbulence	335
11.1.1	Isotropy	336

11.1.2	Momentum and Kinetic Energy in Homogeneous Turbulence	338
11.1.3	Large and Small Length Scales	341
11.1.4	Equilibrium Range and Inertial Subrange	344
11.2	Turbulent Mixing	346
11.2.1	Heuristic Approaches	347
11.2.2	Turbulent Diffusion in the Lagrangian Reference System	350
11.2.3	Eulerian Diffusion by Small-Scale Turbulence	350
11.3	Inhomogeneous Three-Dimensional Turbulence	352
11.3.1	Energetic Constraints	353
11.3.2	Turbulence Models for the Surface Boundary Layer	355
11.3.3	Turbulence in the Ocean Interior	363
12	Geostrophic Turbulence	367
12.1	Homogeneous Turbulence in Two Dimensions	368
12.1.1	Inverse Energy Cascade	369
12.1.2	A Numerical Example of Two-Dimensional Turbulence	370
12.1.3	Equilibrium Range	373
12.2	Mesoscale Eddies and Their Impact on the Mean Flow	375
12.2.1	Energetics of Mesoscale Eddies and the Lorenz Cycle	375
12.2.2	Isopycnal Mixing Tensor	383
12.2.3	Transformed Eulerian Mean	385
12.2.4	Gent and McWilliams Parameterization and the Bolus Velocity	387
12.2.5	Isopycnal Mixing and Transformed Eulerian Mean	388
12.2.6	* Mesoscale Eddy Effects in the Momentum Equation	391
12.3	* Alternative Averaging Frameworks	396
12.3.1	Temporal Residual Mean	399
12.3.2	Rotational Eddy Fluxes	404
12.3.3	Generalized Osborn–Cox Relation	410
12.3.4	Generalized Lagrangian Mean	413
12.3.5	Semi-Lagrangian (Isopycnal) Mean	415
12.3.6	Relating Lagrangian, Eulerian, and Semi-Lagrangian Mean	423

Part V Aspects of Ocean Circulation Theory

13	Forcing of the Ocean	429
13.1	Bulk Formulae as Boundary Conditions	431
13.2	Simplified Boundary Conditions	439
14	The Wind-Driven Circulation	441
14.1	The Flat-Bottom Wind-Driven Circulation	443
14.1.1	The Elementary Current System	444

14.1.2	Ekman Spiral	446
14.1.3	Ekman Transport	448
14.1.4	Ekman Pumping	449
14.1.5	Equilibrium Wind-Driven Model Regimes	451
14.1.6	The Western Boundary Current	456
14.2	The Role of Stratification and Topography	459
14.2.1	The JEBAR Term	460
14.2.2	The f/h Contours	461
14.2.3	Sverdrup's Catastrophe	462
14.2.4	The Bottom Pressure Torque	464
14.2.5	A Realistic Application of the BARBI Model	466
14.2.6	The Baroclinic Stommel Equation	468
14.3	Main Thermocline Dynamics	472
14.3.1	Scaling Considerations	473
14.3.2	Similarity Solutions	474
14.3.3	Ideal Fluid Solutions	475
14.3.4	Thermocline Ventilation in an Isopycnal Layer Model	476
14.3.5	Circulation in Unventilated Regions	481
15	The Meridional Overturning of the Oceans	483
15.1	Basic Ingredients of the Meridional Overturning	487
15.1.1	Water Masses of the Ocean	487
15.1.2	The Thermohaline Surface Forcing	488
15.1.3	The Asymmetry of the Meridional Overturning	490
15.1.4	The Formation of Water Masses	491
15.2	The Stommel–Arons Overturning Model	494
15.3	Sandström's Inference	499
15.3.1	Consequences from Bjerknes' Theorem	500
15.3.2	Thermodynamics of the Meridional Overturning	501
15.3.3	Energetics of the Meridional Overturning	502
15.4	Scaling Laws for the Meridional Overturning	504
15.4.1	Conventional Scaling Attempts	505
15.4.2	A Frictional Model of the Meridional Overturning	507
15.5	Box Models of the Meridional Overturning	510
15.5.1	Motivation and Construction of Box Models	511
15.5.2	Stommel's Box Model	512
15.5.3	Welander's Box Model	521
15.5.4	An Inconsistency of the Box Models	525
15.5.5	A Box Model with Forcing Induced by Wind and Eddies ...	530
15.6	Zonally Averaged Models of the Meridional Overturning	532
15.6.1	The Zonally Averaged System of Equations	533
15.6.2	The Downgradient Closures	534
15.6.3	A Vorticity-Based Closure	537
15.6.4	A Zonally Averaged Model with Consistent Dynamics	539

15.6.5	Zonally Averaged Models Versus Box Models	541
15.6.6	* A Low-Order Model of the Meridional Overturning	545
16	The Circulation of the Southern Ocean	557
16.1	Basic Ingredients of Southern Ocean Dynamics	558
16.1.1	The Antarctic Circumpolar Current	558
16.1.2	Mesoscale Eddies	559
16.1.3	The Meridional Overturning Circulation	560
16.2	Homogeneous Wind-Driven Models of the Southern Ocean	562
16.2.1	A Homogeneous Model with a Flat Bottom and Bottom Friction	565
16.2.2	Hidaka's Dilemma with Lateral Friction	567
16.2.3	Homogenous Southern Ocean with Topography	569
16.2.4	The Barotropic Circulation over Closed f/h Contours	574
16.3	The Meridional Overturning of the Southern Ocean	581
16.3.1	The Eulerian Meridional Overturning Circulation	581
16.3.2	The Isopycnal Overturning Circulation	584
16.3.3	Interfacial Formstress and Vertical Transfer of Momentum	585
16.3.4	The Residual Overturning Circulation	587
16.4	The Zonal Mean Dynamics	589
16.4.1	Complete Balance of Zonal Mean Zonal Momentum	590
16.4.2	Barotropic and Baroclinic Bottom Formstress	594
16.5	The Vertically Averaged Dynamics of the Southern Ocean	594
16.5.1	The Vertically Averaged Momentum Budget	594
16.5.2	The Vertically Averaged Vorticity Budget	596
16.5.3	The Baroclinic Stommel Model of the ACC	598
16.6	Simple Models of the Zonally Averaged Southern Ocean	599
16.6.1	A Model of Vanishing Residual Overturning Circulation	600
16.6.2	Models Driven by Surface Windstress and Thermohaline Forcing	602
16.7	* Simple Models with Bottom Formstress	612
16.7.1	A Homogeneous Charney–DeVore Model	613
16.7.2	A Baroclinic Charney–DeVore Model	616

Part VI Appendix

A	Mathematical Basics	627
A.1	Representation of Hydrodynamic Fields	627
A.1.1	Scalar and Vector Fields	627
A.1.2	Divergence and Gauss' Integral Theorem	629
A.1.3	Rotation and Stokes' Integral Theorem	629
A.1.4	Velocity Potential and Stream Function	631
A.1.5	Integral Theorems in Two Dimensions	632

A.1.6	Tensor Fields	632
A.1.7	Gradient Formation in a Vector Field.....	634
A.1.8	Some Useful Differential and Integral Formulas	634
A.2	Differential Equations	635
A.2.1	Dynamical Systems: Fix Soints, Stability, and Bifurcations .	635
A.2.2	Boundary Value Problems	642
A.3	Description of Random Fields	647
A.3.1	Random Variables	647
A.3.2	Random Functions	649
A.4	Coordinate Systems	652
A.4.1	General Curvilinear Orthogonal Coordinates	653
A.4.2	Oblate Spheroidal Coordinates	655
A.4.3	Spherical Coordinates	656
B	Models of the Ocean Circulation	659
B.1	Models Based on Isopycnal Coordinates	659
B.1.1	Equations of Motion in Isopycnal Coordinates	659
B.1.2	Layer Models	661
B.1.3	A Two-Layer Quasi-Geostrophic Model	662
B.1.4	A Planetary-Geostrophic Isopycnal Layer Model.....	664
B.2	BARBI: A Model of the Wind-Driven Circulation	665
B.2.1	Derivation of BARBI	665
B.2.2	Eddy Parameterization for the BARBI Model	670
B.2.3	The Closure of the Infinite Hierarchy	672
B.2.4	The Two-Mode BARBI Model	673
B.2.5	Comparison of BARBI and QG	674
B.2.6	Waves in BARBI	675
B.3	Spectral Models	678
B.4	Box Models	679
	References	683
	Index	697

Chapters and sections with an asterisk indicate material that is not written for a beginner and may be omitted in a first reading.