

Contents

Summary	v
1 Introduction	1
2 Literature Review	7
2.1 Reachability Analysis of Stochastic Hybrid Systems	7
2.2 Set-Representation	11
2.3 Contribution of this thesis	17
3 Theoretical Background	19
3.1 Stochastic Hybrid System	20
3.2 Set Representation	20
3.3 Multivariate Random Distributions	25
3.4 Semidefinite Programming	28
4 Robust Control of Affine Systems with Reachable Set Computation	31
4.1 Reachability Problem of Discrete-Time Affine Systems	33
4.2 Control Law Specification	35
4.3 Algorithmic Solution Based on Semi-Definite Programming	37
4.4 Nonlinear Systems	46
4.4.1 Conservative Linearization Procedure	48
4.4.2 Extension of Control Algorithm for Affine Systems	51
4.5 Numerical Example	56
4.6 Discussion	60
5 Probabilistic Reachable Set Computation for Controller Synthesis	63
5.1 Affine Probabilistic Systems	65
5.1.1 System Definition and Probabilistic Reachable Computations	65
5.1.2 Definition of the Set-To-Target Control Problem for APS	68
5.2 Review of Methods for the Approximation of Chance Constraints	70
5.2.1 Set-Based Evaluation of Chance Constraints	70
5.2.2 Locally Linear Approximation of Multivariate CDF's	70
5.2.3 Scenario-Based handling of Chance Constraints	75
5.3 Controller Synthesis for Affine Probabilistic Systems	78
5.3.1 Semi-Definite Programming for Continuous Input	78
5.3.2 Algorithmic Solution Procedure	80

5.4	Illustration of the Control Algorithm	83
5.4.1	System Model	83
5.4.2	Discussion of the different results	85
5.4.3	General Evaluation of the Chance Constraint Approximation	88
5.5	Discussion of the Controller Synthesis for Affine Probabilistic Systems	89
6	Hybrid Controller Synthesis for Switched Affine Probabilistic Systems	93
6.1	Switched Affine Probabilistic Systems	94
6.1.1	System Definition and Probabilistic Reachable Computations	95
6.1.2	Definition of the Set-To-Target Control Problem for SAPS	96
6.2	Discrete Optimization with embedded SDP for Controller Synthesis	98
6.2.1	Tree Search for the Discrete Input	98
6.2.2	Synthesis of the Continuous Control Law via SDP	102
6.3	Exemplary Application of Tree-Search-Heuristics	106
6.3.1	Generic System Model of an SAPS	106
6.3.2	Discussion of the Impact of the Tree Search Heuristic	108
6.4	Discussion	111
7	Control of Piece-Wise Affine Probabilistic Systems	113
7.1	Piece-Wise Affine Probabilistic Systems	113
7.1.1	System Definition and Probabilistic Reachable Computations	114
7.1.2	Definition of the Set-To-Target Control Problem PWAPS	116
7.2	Controller Synthesis for PWAPS	117
7.2.1	Push, Branch, and Merge Procedure	119
7.2.2	Algorithmic Synthesis of the Continuous Control Law	129
7.3	Numerical Example	132
7.3.1	Exemplary Model of an PWAPS	132
7.3.2	Discussion of the Numerical Results	133
7.4	Discussion of the Method	135
8	Conclusion	137
	List of Symbols	141
	References	151