

Contents

Preamble: a polemical introduction to the stable category	1
I. The equivariant stable category	6
§1. Recollections about equivariant homotopy theory	8
§2. Categories of G -prespectra and G -spectra	11
§3. The functors E_*X , $F(X,E)$, E/H , and E^H ; homotopy theory	16
§4. The functors $A^Z\Gamma^\infty$; sphere spectra and homotopy groups	21
§5. G -CW spectra and the stable category	27
§6. The stable category, cohomology, and the cylinder construction	32
§7. Shift desuspension and weak equivalence	39
§8. Special kinds of G -prespectra and G -spectra	48
II. Change of universe, smash products, and change of groups	54
§1. Change of universe functors	57
§2. Families and change of universe isomorphisms	62
§3. Smash products and function spectra	68
§4. Change of groups functors and isomorphisms	75
§5. Space level constructions	84
§6. A generalization of Wirthmüller's isomorphism	88
§7. A generalization of Adams' isomorphism	96
§8. Coherent families of equivariant spectra	102
§9. The construction of (G/N) -spectra from G -spectra	107
III. Equivariant duality theory	117
§1. Categorical duality theory	119
§2. Duality for G -spectra	128
§3. Slant products and V -duality of G -spaces	135
§4. Duality for compact G -ENR's	142
§5. Duality for smooth G -manifolds	152
§6. The equivariant Poincaré duality theorem	157
§7. Trace maps and their additivity on cofibre sequences	160
§8. Space level analysis of trace maps	169
IV. Equivariant transfer	175
§1. Types of equivariant bundles	178
§2. The pretransfer	181
§3. The definition and axiomatic properties of the transfer	186
§4. The behavior of the transfer with respect to change of groups	191
§5. Product and Euler characteristic formulas	196
§6. The sum decomposition and double coset formulas	203
§7. Transitivity relations	212

§8. Cohomological transports	217
§9. Classification of transforms and uniqueness of transfers	227
V. The Burnside ring and splittings in equivariant homology theory	236
§1. Equivariant Euler characteristics	239
§2. The Burnside ring and $\pi_G^0(S)$	245
§3. Prime ideals in $A(G)$	251
§4. Idempotent elements in $A(G)$	254
§5. Localizations of $A(G)$ and of $A(G)$ -modules	259
§6. Localizations of equivariant homology and cohomology theories	267
§7. Preliminaries on universal $(\mathcal{E}', \mathcal{E})$ -spaces and adjacent pairs	272
§8. Concentration of homology and cohomology theories between families	277
§9. Equivariant stable homotopy groups and Mackey functors	283
§10. Normal subgroups in equivariant stable homotopy theory	290
§11. Fixed point spectra of suspension spectra	293
VI. Twisted half smash products and extended powers	299
§1. Statements of results about $\chi \ltimes E$	301
§2. Constructions of $\chi \ltimes E$; proofs	309
§3. Relations between smash products and twisted half smash products	323
§4. Untwisting G -homotopies and π -actions	332
§5. Extended powers of G -spectra	344
VII. Operad ring spectra	350
§1. Operads and extended powers	351
§2. Actions of operads on spectra	361
§3. The constructions CX and CE	368
§4. Pairings and operad actions on CE	374
§5. Splitting theorems and James maps	379
VIII. The homological analysis of extended powers	384
§1. Cellular chains and filtered spectra	384
§2. Spectral sequences and cellular chains of extended powers	391
§3. Steenrod operations in $D_{\pi}E$	400
IX. Thom spectra	407
§1. Preliminaries on sphere spaces and spherical fibrations	408
§2. Preliminaries on \mathcal{J} -spaces	415
§3. The definition and basic examples of Thom spectra	420
§4. Invariance properties of Thom spectra	426
§5. The Thom isomorphism	433

§6. Extended powers of Thom spectra	439
§7. Thom spectra and operad ring spectra	443
X. Equivariant Thom spectra	450
§1. Preliminaries on G -vector bundles	451
§2. Preliminaries on $G\mathbb{S}$ -spaces	452
§3. The definition and basic properties of Thom G -spectra	459
§4. Homotopy invariance properties of Thom G -spectra	465
§5. The equivariant Thom isomorphism	466
§6. Twisted half-smash products and Thom G -spectra	472
Appendix: Analysis of the passage from prespectra to spectra	475
§1. The construction of the functor L	475
§2. The behavior of L with respect to limits	481
§3. Prespectrum and spectrum level closed inclusions	486
§4. The point-set topology of CW -spectra	490
Bibliography	496
Index	504
Index of notations -- Roman letter	523
Index of notations -- Greek letter	527
Index of notations -- non-alphabetic, subscripts and superscripts	531
Index of categories	532
Index of adjoint pairs of functors	533
Index of natural transformations	535