

Contents

1. Gravitational Waves in Strong Gravitational Fields	1
1.1 Formalism of Complex Null Tetrads and Petrov Classification of Algebraic Types of the Weyl Tensor	1
1.2 Gravitational Waves and Generalized Solutions of the Equations of the Electrovacuum	17
1.2.1 Properties of Families of Isotropic Geodesics in General Relativity	20
1.2.2 Propagation of Breaks in the Gravitational Field and Their Algebraic Classification	23
1.2.3 Decay of an Arbitrary Break in the Vacuum Gravitational Field	30
1.2.4 The Interaction of Short Gravitational and Electromagnetic Waves in Arbitrary External Electromagnetic Fields	32
1.2.5 Algebraic Structure of Perturbations of the Weyl Tensor in the Case of High-Frequency Waves	39
1.2.6 Behavior of Short-Wave Perturbations of the Gravitational Field Near Caustic Surfaces	40
1.3 Interaction of Gravitational and Electromagnetic Waves	45
1.3.1 Curvature of Space-Time in a Plane Electromagnetic Wave	45
1.3.2 Nonlinear Interaction of Plane Waves	48
1.3.3 Propagation of Weak Electromagnetic and Gravitational Waves in the Field of a Strong Electromagnetic Wave	52
1.3.4 Oscillatory Character of Solutions Near a Singularity	56
1.4 Conditions on Surfaces with Strong Breaks in Theories of the Gravitational Field	59
2. The Classical Theory of Black Holes	65
2.1 Asymptotically Flat Gravitational Fields	70
2.2 Basic Elements of the Theory of Lie Groups and Exterior Forms	76
2.2.1 The Concept of Lie Groups	76
2.2.2 The Concept of Skew and Differential Forms	78
2.2.3 Frobenius's Theorem	80
2.3 Stationary Gravitational Fields	82

2.4	Energetics of Black Holes	103
2.4.1	Temperature of a Black Hole	103
2.4.2	Electrostatic Potential of a Black Hole	104
2.4.3	Formula for the Mass of a Black Hole	105
2.4.4	"Thermodynamics" of Black Holes	107
3.	Stationary Axially Symmetric Fields in General Relativity	113
3.1	Canonical Equations of Massless Fields Admitting Abelian Two-Parameter Groups of Motions	114
3.2	Infinite-Dimensional Algebra and Lie Group of the Equations for the Neutrino Electrovacuum	125
3.3	General Solution of the Einstein-Maxwell Equations for Ernst Data Regular Locally on the Symmetry Axis	137
3.4	Lie-Bäcklund Groups of Integrable Systems of Mathematical Physics	157
4.	Propagation of Waves in the Gravitational Fields of Black Holes	168
4.1	Propagation of Short Waves in the Field of a Charged Black Hole	170
4.1.1	Short Waves in the Nordström-Reissner Field	170
4.1.2	Short Waves in the Neighborhood of a Rotating Charged Black Hole	174
4.2	Asymptotic Theory of Scattering of Wave Packets in the Gravitational Field of a Black Hole	176
4.3	Wave Fields Outside a Collapsing Star	184
4.3.1	Derivation of the Basis Equations	185
4.3.2	Boundary Conditions and General Properties of the Reflection and Transmission Coefficients of Waves	189
4.3.3	Properties of Radiation Emitted by a Collapsing Body Near the Horizon	192
4.3.4	Behavior of the Transmission Coefficient for Small ω	193
4.3.5	Laws of Attenuation of the "Tails" of the Multipole Radiation	199
5.	Relativistic Hydrodynamics	202
5.1	Relativistic Dynamics of a Point and Gas of Free Particles ..	203
5.2	Thermodynamic Equilibrium in an Ideal Gas	205
5.3	Relativistic Dynamics and Acoustics of an Ideal Gas	212
5.3.1	Shock Waves and Vortex Motions of an Ideal Gas ...	214
5.3.2	Potential Motions	215
5.3.3	Acoustic Waves in a Relativistic Gas	216
5.3.4	Nonlinear Acoustics of an Expanding Universe: Relativistic Theory	218
5.4	Relativistic Magnetohydrodynamics	221

5.4.1	Shock Waves in Magnetohydrodynamics and the Hugoniot Adiabats	223
5.4.2	Properties of MHD Breaks	225
5.4.3	Relative Positions of the Poisson and Hugoniot Adiabats	227
5.5	Hydrodynamical Flow Resulting from Production of Ultrarelativistic Particles in the Field of a Black Hole	229
5.6	Self-Similar Motions of an Ultrarelativistic Gas with Spherical or Cylindrical Symmetry	231
5.6.1	Qualitative Investigation of (5.6.5)	233
5.6.2	Ejection of Matter from a Singular Point (Axis) at the Instant $t = 0$	235
5.6.3	Solution of the Cauchy Problem	236
6.	Some Problems of the Dynamics of Waves in Relativistic Cosmology	239
6.1	Development of Inhomogeneities in Models of the Universe with a Cosmological Magnetic Field	243
6.1.1	Unperturbed Solution	244
6.1.2	Notation for Small Perturbations and Coordinate Restrictions	244
6.1.3	Equations of Conservation of Energy-Momentum and of the Magnetic Induction	246
6.1.4	A Closed System for the Odd Perturbations	247
6.1.5	A Closed System for the Even Perturbations	248
6.1.6	Exact Solutions of the Linearized Equations Corresponding to Perturbations of the Coordinate System	248
6.1.7	Closed System of Equations for the Even Perturbations	250
6.1.8	Analysis of the Closed System of Equations for Odd Perturbations	251
6.1.9	Analysis of the Closed System for Even Short-Wave Perturbations	251
6.1.10	Evolution of Perturbations of Arbitrary Finite Scales Near a "Pancake" Singularity	253
6.2	Self-Similar Motions of a Photon Gas in the Friedman-Lemaître Model	255
6.2.1	Derivation of a Closed System of Ordinary Differential Equations and Conditions on Shock Waves	257
6.2.2	Friedman Solution and Qualitative Investigation of the System of Equations for $x(\zeta)$ and $V(\zeta)$	259
6.2.3	Discussion of the Results	264
7.	Acoustic Phenomena in Strong Gravitational and Magnetic Fields	269
7.1	Propagation of Nonlinear Short Acoustic Waves	271
7.1.1	Derivation of the Model Equations	271

7.1.2	The Energy Density, Enthalpy Flux Vector, and Momentum Density in an Acoustic Wave Traveling Through an Arbitrary Background	275
7.1.3	Distortion of Short Acoustic Waves	277
7.1.4	Reflection of Acoustic Waves from Strong Breaks ...	279
7.2	The Form of the Magnetosphere in a Nonuniform Plasma ...	282
7.2.1	The Form of the Static Magnetosphere in a Nonuniform Plasma	282
7.2.2	Shape of the Magnetosphere of a Star Rotating in the Supersonic Regime	289
7.3	Generation of Acoustic Waves by the Rotating Magnetosphere in the Stellar Wind	295
7.3.1	Acoustic Waves in an Isothermal Atmosphere	297
7.3.2	Generation of Short Waves	299
7.3.3	Inversion of Acoustic Waves	300
7.3.4	Heating of the Plasma Due to Dissipation of Acoustic Waves	302
7.3.5	Inclusion of Viscosity of the Gas in Calculating the Torque Acting on the Rotating Magnetosphere ...	303
7.3.6	Supersonic Rotation of a Pulsar	312
7.3.7	A Model Problem of Stationary Accretion of a Gas with Index of the Adiabatic $\gamma < 5/3$ in the Presence of Energy Release	313
7.3.8	The Stability of Rotation of Pulsars in Close Binary Systems	314
7.4	The Stability of Uniform Nonlinear Pulsations of Gravitating Gaseous Spheres	318
7.5	Nonlinear Transverse Oscillations at Resonance in a Layer of an Ideally Conducting Fluid in a Magnetic Field	326
7.5.1	Magnetohydrodynamic Analogy of One-Dimensional Motions of a Nonlinearly Elastic Body with Plane Waves	326
7.5.2	Derivation of Basis Equations for Oscillations Near Resonance	329
7.5.3	Investigation of Oscillations in an Elastic Layer	332
7.6	Excitation of Shock Waves in a Layer of an Ideally Conducting Gas at Resonance in a Magnetic Field	336
7.6.1	Formulation of the Problem	337
7.6.2	Investigation of the Form of the Oscillations Near a Resonance	339
	References	349
	Subject Index	361