

Contents

1. Introduction	1
2. Variety in Structures	5
2.1 Crystals	5
2.2 Incommensurate and Long-Period Structures	6
2.3 Quasicrystals	7
2.3.1 Quasiperiodicity	8
2.3.2 2D Quasicrystalline Tilings	11
2.3.3 A Brief Recapitulation	13
2.3.4 3D Quasicrystals	15
2.4 Liquid Crystals	16
2.5 Glass	17
2.6 Systems with Quasi Long-Range Order	19
2.7 Overview	20
3. Order Out of Disorder	22
3.1 Landau Theory	22
3.1.1 Transition in a System with a Scalar Order Parameter	22
3.1.2 Role of Symmetry	23
3.1.3 Systems with a Complex Order Parameter	24
3.1.4 Order Parameter Space	25
3.1.5 Generalized Landau Expansion	25
3.1.6 Fluctuations	26
3.2 Conjugate Field	26
3.3 Symmetry Breaking: Further Aspects	27
3.4 Goldstone Modes	29
3.5 Generalized Rigidity	30
3.6 Quasi LRO	30
3.7 Overview	32
4. Defects and Topology	34
4.1 Basic Strategy	34
4.2 Some Basic Concepts of Topology	36
4.3 Continuous Groups and Topological Spaces	37
4.4 The First or the Fundamental Homotopy Group and Defects	38
4.4.1 Burgers Circuit	38

4.4.2 Closure Misfit	39
4.4.3 Fundamental Group	40
4.4.4 Π_1 (V) and Defects	40
4.5 Some Examples	41
4.6 Stability	42
4.7 Combination of Defects	43
4.8 Other Homotopy Groups	44
4.9 Ordered Media with Broken Translational Symmetry	45
4.10 Summary	46
5. Structures by Projection	48
5.1 Concerning Tilings	49
5.2 Regular Polytopes	51
5.3 Amorphous Structures from Mappings of Polytopes	51
5.4 Line Defects in Amorphous Structures	52
5.5 Disclinations and Frank-Kasper Chains	54
5.6 Mapping from S^3 to E^3	55
5.7 Defects and Star Mapping	57
5.8 Mapping by Disclination Procedure	57
5.9 Decoration	60
5.10 Defects in the CRN	61
5.11 Amorphous Structures by Projection of Hyperbolic Tilings ..	61
5.12 Polymers and Polytopes	62
5.13 Quasicrystals by the Projection Method	62
5.13.1 Generation of the Penrose Chain	62
5.13.2 Varying the Choice of the Unit Cell	63
5.13.3 Role of the Slope	63
5.13.4 Effect of Translating \mathcal{S} Laterally	63
5.13.5 Role of the Orientation of ξ	64
5.14 Generalization	64
5.15 Some Comments on the Projection Method	66
5.16 Miller Indices for Quasicrystals	66
5.17 Diffraction Patterns of Quasicrystals	67
5.18 Incommensurate Crystals	69
5.19 Summary	71
6. Beyond Simple Geometry	73
6.1 Some Basics	73
6.2 Landau Theory and Ordered Atomic Structures	74
6.2.1 Free Energy Expansion	74
6.2.2 Liquid-Solid Transition	76
6.2.3 BCC Versus Icosahedral Ordering	77
6.3 Orientational Ordering	79
6.4 Orientational Order Versus Translational Order	81
6.5 Landau Theory and Amorphous Structures	82

6.6	Landau Theory and Liquid Crystals	86
6.6.1	Liquid-to-Nematic Transition	86
6.6.2	Deformation Energy	88
6.6.3	Nematic-to-Smectic A (NA) Transition	89
6.6.4	Defects in Smectic A	89
6.6.5	Analogies to the Superconductor	90
6.7	Hydrodynamics	92
6.8	Fluctuations and the Landau Theory	94
6.9	Frustration and the Disruption of Order	95
6.10	Defect-Dominated Structures	97
6.10.1	Role of Topological Defects	97
6.10.2	Topological Order	99
6.10.3	Critical Behaviour of the Model	100
6.10.4	Disclinations and 2D Melting	101
6.10.5	Landau Theory and Defect-Mediated Transitions	102
6.11	Overview	104
7.	Tilings in One Dimension	106
7.1	Structures and Competing Periodic Potentials	106
7.1.1	The Problem	106
7.1.2	Structures and Maps	107
7.1.3	Trajectories and Structures	109
7.2	Portrait of the Penrose Chain	110
7.3	Spatial Chaos and Amorphous Structures	112
7.4	Summary	116
8.	Ergodicity Breaking	117
8.1	Basic Ideas	117
8.2	Time Scales and Broken Ergodicity	117
8.3	Broken Ergodicity and Symmetry Breaking	119
8.4	The Spin Glass	120
8.5	The Case of Glass	123
8.6	Generalization	124
9.	Symmetry Breaking – A Second Look	125
9.1	Orbits and Strata in Crystal Physics	125
9.2	Symmetry Breaking and Strata	126
9.3	Isotropy Subgroups of the Euclidean Group $E(3)$	127
9.4	More About Extensions to $E(3)$	129
9.5	Patterns in Nonequilibrium Systems	130
9.6	Cylindrical Crystallography	131
	Appendix: Special Topics	133
A.	Hydrodynamics	133
A.1	Hydrodynamic Equations	135

A.2 Ordered Media with Continuous Broken Symmetries	146
A.2.1 Hydrodynamics of a Solid	146
A.3 The Poisson Bracket Method in Hydrodynamics	149
A.4 Summary	153
B. Curved Space and Parallel Transport of Vectors	154
B.1 Parallel Transport of Vectors	154
B.2 The Covariant Derivative	157
B.3 The Curvature	158
B.4 The Torsion	158
B.5 Mapping from Curved Space to Flat Space	159
C. DRP Structures, Polytopes and the Tiling of S^3	160
D. Some Aspects of Group Theory	162
D.1 Group Morphisms	162
D.2 Transformation Group, Group Action and Orbits	163
E. A Brief Introduction to Homotopy and Lie Groups	166
E.1 Topology	166
E.2 Elements of Homotopy Theory	168
E.2.1 The First Homotopy Group	169
E.2.2 Higher Homotopy Groups	171
E.3 Continuous Groups and Lie Groups	172
F. Local Gauge Invariance and Gauge Theories	181
F.1 Internal Connection	181
F.2 Gauge Field Theory	182
F.3 $U(1)$ Gauge Symmetry	183
F.4 Non-Abelian Gauge Groups	185
F.5 Gauge Theory of Dislocations and Disclinations	187
References	193
Author Index	201
Subject Index	203