

Contents

1. Introduction	1
1.1 The Fermi-Pasta-Ulam Problem	1
1.2 Hénon-Heiles Calculation	5
1.3 Discovery of Solitons	8
1.4 Dual Systems	10
2. The Lattice with Exponential Interaction	14
2.1 Finding of an Integrable Lattice	14
2.2 The Lattice with Exponential Interaction	16
2.3 Periodic Solutions	19
2.4 Solitary Waves	22
2.5 Two-Soliton Solutions	25
2.6 Hard-Sphere Limit	29
2.7 Continuum Approximation and Recurrence Time	31
2.8 Applications and Extensions	34
2.9 Poincaré Mapping	36
2.10 Conserved Quantities	38
3. The Spectrum and Construction of Solutions	42
3.1 Matrix Formalism	42
3.2 Infinite Lattice	46
3.3 Scattering and Bound States	50
3.4 The Gel'fand-Levitan Equation	55
3.5 The Initial Value Problem	58
3.6 Soliton Solutions	60
3.7 The Relationship Between the Conserved Quantities and the Transmission Coefficient	66
3.8 Extensions of the Equations of Motion and the Kac-Moerbeke System	71
3.9 The Bäcklund Transformation	76
3.10 A Finite Lattice	84
3.11 Continuum Approximation	93
4. Periodic Systems	98
4.1 Discrete Hill's Equation	98
4.2 Auxiliary Spectrum	106
4.3 Relation Between $\mu_j(k)$ and $\mu_j(0)$	110

4.4	Related Integrals on the Riemann Surface	115
4.5	Solution to the Inverse Problem	123
4.6	Time Evolution	125
4.7	A Simple Example (A Cnoidal Wave)	130
4.8	Periodic System of Three-Particles	142
5.	Application of the Hamilton-Jacobi Theory	147
5.1	Canonically Conjugate Variables	147
5.2	Action Variables	154
6.	Recent Advances in the Theory of Nonlinear Lattices	159
6.1	The KdV Equation as a Limit of the TL Equation	159
6.2	Interacting Soliton Equations	161
6.3	Integrability	163
6.4	Generalization of the TL Equation	165
6.5	Two-Dimensional TL	166
6.6	Bethe Ansatz	168
6.7	The Thermodynamic Limit	171
6.8	Hierarchy of Nonlinear Equations	173
6.9	Some Numerical Results	174
Appendices		177
A	Stieltjes' Method for a Continued Fraction	177
B	Suite de Sturm	184
C	Simple Roots of $\Delta^2(\lambda) = 4$ Determine all the Roots	184
D	All the Auxiliary Spectrum μ_j are Simple	187
E	Lagrange's Interpolation Formula	188
F	Multivariable ϑ Function (Riemann's ϑ Function)	189
G	Hirota's Method	192
H	Induction Phenomenon	196
I	Partition Function	201
J	Dispersion Relation for the Lattice Without Expansion	203
Simplified Answers to Main Problems		207
References		213
Bibliography		219
Subject Index		221
List of Authors Cited in Text		225