

Contents

1	Introduction	1
1.1	Literature Review	5
1.2	Solution Methods	7
1.3	Approximate Methods	9
1.4	Review of Analytical-Numerical Methods in Moving Load Problems	12
1.4.1	d'Alembert Method	13
1.4.2	Fourier Method	14
1.4.3	Lagrange Formulation	17
1.5	Examples	18
2	Analytical Solutions	21
2.1	A Massless String under a Moving Inertial Load	22
2.1.1	Case of $\alpha \neq 1$	23
2.1.2	Case of $\alpha = 1$	25
2.2	Discontinuity of the Solution	26
2.3	Conclusions	29
3	Semi-analytical Methods	31
3.1	String	32
3.1.1	Fourier Analysis	32
3.1.2	The Lagrange Equation	37
3.2	Bernoulli–Euler Beam	46
3.2.1	Fourier Solution	47
3.2.2	The Lagrange Equation of the Second Kind	50
3.2.3	Conclusions	55
3.3	Timoshenko Beam	55
3.3.1	Fourier Solution	56
3.3.2	The Lagrange Equation	56
3.3.3	Examples	59
3.3.4	Conclusions and Discussion	61

3.4	Bernoulli–Euler Beam vs. Timoshenko Beam	66
3.5	Plate	67
3.6	The Renaudot Approach vs. The Yakushev Approach	70
3.6.1	The Renaudot Approach	71
3.6.2	The Yakushev Approach	72
4	Review of Numerical Methods of Solution	77
4.1	Oscillator	79
4.1.1	String Vibrations under a Moving Oscillator	79
4.1.2	Beam Vibrations under a Moving Oscillator	83
4.2	Inertial Load	84
4.2.1	A Bernoulli–Euler Beam Subjected to an Inertial Load	85
4.2.2	A Timoshenko Beam Subjected to an Inertial Load	89
5	Classical Numerical Methods of Time Integration	95
5.1	Integration of the First Order Differential Equations	97
5.2	Single-Step Method SSPj	102
5.3	Central Difference Method	105
5.3.1	Stability of the Method	107
5.3.2	Accuracy of the Method	108
5.4	The Adams Methods	109
5.4.1	Explicit Adams Formulas (Open)	110
5.4.2	Implicit Adams Formulas (Closed)	112
5.5	The Newmark Method	114
5.6	The Bossak Method	117
5.7	The Park Method	118
5.8	The Park–Housner Method	118
5.8.1	Stability of the Park–Housner Method	119
5.9	The Trujillo Method	121
6	Space–Time Finite Element Method	123
6.1	Formulation of the Method—Displacement Approach	129
6.1.1	Space–Time Finite Elements in the Displacement Description	135
6.2	Properties of the Integration Schemes	138
6.2.1	Accuracy of Methods	140
6.3	Velocity Formulation of the Method	140
6.3.1	One Degree of Freedom System	140
6.3.2	Discretization of the Differential Equation of String Vibrations	144
6.3.3	General Case of Elasticity	149
6.3.4	Other Functions of the Virtual Velocity	151
6.4	Space–Time Element Method and Other Time Integration Methods	154

6.4.1	Convergence	154
6.4.2	Phase Error	157
6.4.3	Non-inertial Problems	158
6.5	Space–Time Finite Element Method vs. Newmark Method	160
6.6	Simplex Elements	161
6.6.1	Property of Space Division	162
6.6.2	Numerical Efficiency	167
6.7	Simplex Elements in the Displacement Description	169
6.7.1	Triangular Element of a Bar Vibrating Axially	169
6.7.2	Space–Time Finite Element of the Beam of Moderate Height	170
6.7.3	Tetrahedral Space–Time Element of a Plate	172
6.8	Triangular Elements Expressed in Velocities	176
7	Space–Time Finite Elements and a Moving Load	181
7.1	Space–Time Finite Element of a String	182
7.1.1	Discretization of the String Element Carrying a Moving Mass	182
7.1.2	Numerical Results	184
7.1.3	Conclusions	188
7.2	Space–Time Elements for a Bernoulli–Euler Beam Carrying a Moving Mass	188
7.2.1	Numerical Results	190
7.3	Space–Time Element of Timoshenko Beam Carrying a Moving Mass	198
7.3.1	Conclusions	203
7.4	Space–Time Finite Plate Element Carrying a Moving Mass	204
7.4.1	Thin Plate	204
7.4.2	Thick Plate	213
7.4.3	Plate Placed on an Elastic Foundation	215
7.5	Problems with Zero Mass Density	218
8	The Newmark Method and a Moving Inertial Load	223
8.1	The Newmark Method in Moving Mass Problems	223
8.2	The Newmark Method in the Vibrations of String	226
8.3	The Newmark Method in Vibrations of the Bernoulli–Euler Beam	229
8.4	The Newmark Method in Vibrations of a Timoshenko Beam	230
8.5	Numerical Results	230
8.6	Accelerating Mass—Numerical Approach	233
8.6.1	Mathematical Model	233
8.6.2	The Finite Element Carrying the Moving Mass Particle ...	235
8.6.3	Accelerating Mass—Examples	238
8.7	Conclusions	239

9	Meshfree Methods in Moving Load Problems	241
9.1	Meshless Methods (Element-Free Galerkin Method)	241
9.2	Results	243
10	Examples of Applications	247
10.1	Dynamics of the Classical Vehicle–Track System	249
10.2	Dynamics of the System Vehicle—Y-Type Track	253
10.3	Dynamics of Subway Track	262
10.4	Vibrations of Airport Runways	266
	Appendix	271
A	Computer Programs	271
A.1	String—Space–Time Element Method	271
A.2	Timoshenko Beam—Newmark Method	274
A.3	Mindlin Plate—Space–Time Element Method	277
A.4	Kirchhoff Plate — Space-Time Element Method	283
	References	285
	Index	293