

Contents

1. Cyclic, Dicyclic and Metacyclic Groups	1
1.1 Generators and relations	1
1.2 Factor groups	2
1.3 Direct products	3
1.4 Automorphisms	5
1.5 Some well-known finite groups	6
1.6 Dicyclic groups	7
1.7 The quaternion group	8
1.8 Cyclic extensions of cyclic groups	9
1.9 Groups of order less than 32	11
2. Systematic Enumeration of Cosets	12
2.1 Coset enumeration	12
2.2 Finding a presentation for a finite group	16
2.3 Finding a presentation for a subgroup	17
2.4 The corresponding permutations	18
3. Graphs, Maps and Cayley Diagrams	18
3.1 Graphs	19
3.2 Maps	20
3.3 Cayley diagrams	21
3.4 Planar diagrams	23
3.5 Unbounded surfaces	24
3.6 Non-planar diagrams	28
3.7 SCHREIER's coset diagrams	31
4. Abstract Crystallography	32
4.1 The cyclic and dihedral groups	33
4.2 The crystallographic and non-crystallographic point groups	33
4.3 Groups generated by reflections	35
4.4 Subgroups of the reflection groups	38
4.5 The seventeen two-dimensional space groups	40
4.6 Subgroup relationships among the seventeen groups	51
5. Hyperbolic Tessellations and Fundamental Groups	52
5.1 Regular tessellations	52
5.2 The Petrie polygon	53
5.3 DYCK's groups	54
5.4 The fundamental group for a non-orientable surface, obtained as a group generated by glide-reflections	56
5.5 The fundamental group for an orientable surface, obtained as a group of translations	58

6. The Symmetric, Alternating, and other Special Groups	61
6.1 ARTIN's braid group	62
6.2 The symmetric group	63
6.3 The alternating group	66
6.4 The polyhedral groups	67
6.5 The binary polyhedral groups	68
6.6 MILLER's generalization of the polyhedral groups	71
6.7 A new generalization	76
6.8 BURNSIDE's problem	80
7. Modular and Linear Fractional Groups	83
7.1 Lattices and modular groups	83
7.2 Defining relations when $n = 2$	85
7.3 Defining relations when $n \geq 3$	88
7.4 Linear fractional groups	92
7.5 The case when $n = 2$ and $q = p$, a prime	93
7.6 The groups $LF(2, 2^m)$	97
7.7 The Hessian group and $LF(3, 3)$	98
7.8 The Mathieu groups	99
8. Regular Maps	101
8.1 Automorphisms	101
8.2 Universal covering	103
8.3 Maps of type $\{4, 4\}$ on a torus	103
8.4 Maps of type $\{3, 6\}$ or $\{6, 3\}$ on a torus	107
8.5 Maps having specified holes	109
8.6 Maps having specified Petrie polygons	111
8.7 Maps having two faces	113
8.8 Maps on a two-sheeted Riemann surface.	115
8.9 Symmetrical graphs	117
9. Groups Generated by Reflections	117
9.1 Reducible and irreducible groups	117
9.2 The graphical notation	118
9.3 Finite groups	119
9.4 A brief description of the individual groups	123
9.5 Commutator subgroups	124
9.6 Central quotient groups	127
9.7 Exponents and invariants	129
9.8 Infinite Euclidean groups	131
9.9 Infinite non-Euclidean groups	132
Tables 1—12	134
Appendix for Chapter 2	143
Bibliography	149
Index	165