

Contents

1.	<i>The Soliton and Its History.</i> By R.K. Bullough and P.J. Caudrey	1
1.1	Russell's Discovery of the 'Great Solitary Wave'	1
1.2	Definition of a Soliton: N-Soliton Solutions of Nonlinear Evolution Equations	5
1.3	Bäcklund Transformations and Conserved Densities	13
1.4	Other Physical Problems and the Discovery of the Inverse Method ..	24
1.5	Operator Pair Formulation of Nonlinear Evolution Equations	29
1.6	Discovery of Some Other N-Soliton Solutions: The AKNS-Zakharov-Shabat 2×2 Scattering Scheme and Its Geometry	32
1.7	Further Progress on Inverse Scattering Methods	48
	References	56
2.	<i>Aspects of Soliton Physics.</i> By G.L. Lamb, Jr. and D.W. McLaughlin (With 5 Figures)	65
2.1	Historical Remarks and Summary	65
2.2	Model Interacting Systems	67
2.2.1	The Interaction of an Electromagnetic Field with a Two-Level Medium	68
2.2.2	Solitons in Plasma Physics	82
2.3	Inverse Spectral Transformation and Motion Invariants	86
2.3.1	Physical Introduction to the Inverse Scattering Method	86
2.3.2	Physical Information from the Inverse Method	91
2.3.3	Hamiltonian Description and Constants of the Motion	93
	Appendix A: Formal Derivation of the Marchenko Equations	100
	References	104
3.	<i>The Double Sine-Gordon Equations: A Physically Applicable System of Equations</i> By R.K. Bullough, P.J. Caudrey, and H.M. Gibbs (With 13 Figures)	107
3.1	Physical Background	107
3.2	Theory of Degenerate SIT	112
3.3	Spin Waves in Liquid ^3He	121
3.4	Perturbation Theory for the Double Sine-Gordon Equation	131
	References	139

4.	<i>On a Nonlinear Lattice (The Toda Lattice).</i> By M. Toda	143
4.1	Nonlinear Lattices	143
4.1.1	Equations of Motion	144
4.1.2	The Dual Transformation	144
4.2	The Exponential Interaction	145
4.2.1	Cnoidal Waves	145
4.2.2	Solitons	146
4.2.3	The Harmonic Limit	147
4.2.4	Two-Soliton Solutions	147
4.3	Matrix Formalism	150
4.3.1	The Inverse Method	150
4.3.2	Multi-Soliton Solutions	152
4.4	The Continuum Limit	152
4.5	Bäcklund Transformations	153
4.6	Concluding Remarks	154
	References	155
5.	<i>Direct Methods in Soliton Theory.</i> By R. Hirota	157
5.1	Preliminaries	157
5.2	Properties of the D Operator	160
5.3	Solutions of the Bilinear Differential Equations	162
5.4	N-Soliton Solution of KdV-Type Equations	164
5.5	Bäcklund Transformations in Bilinear Form	167
	References	175
6.	<i>The Inverse Scattering Transform.</i> By A.C. Newell (With 1 Figure)	177
6.1	General Discussion	177
6.2	The Generalized Zakharov-Shabat Eigenvalue Problem	185
6.3	Evolution of the Scattering Data	188
6.4	The Squared Eigenfunctions and Fourier Expansions	192
6.5	Evolution Equations of Class I	195
6.6	Hamiltonian Structure of the Equations of Class I	198
6.7	Systems with Two Dispersion Relations	201
6.8	Coherent Pulse Propagation	203
6.9	Moving Eigenvalues	206
6.10	The Sine-Gordon Equation	209
6.11	Schrödinger Equation	212
6.12	A Singular Perturbation Theory	217
6.13	Conclusion	230
	References	239

7. <i>The Inverse Scattering Method</i> . By V.E. Zakharov	243
7.1 Preliminary Remarks	243
7.2 The Method of Finding "L-A" Pairs	244
7.3 Elementary Multidimensional Generalisation	249
7.4 Dressing " \hat{L}, \hat{A} " Pairs	253
7.5 The Problem of Reduction and the Physical Interpretation of Examples	258
7.6 Two Dimensional Instability of Solitons	264
7.7 Exact Solutions of Equations in Nonlinear Optics	267
7.8 " $\hat{L}, \hat{A}, \hat{B}$ " Triad	270
7.9 The Conservation of the Spectrum of Operator Families	274
7.10 The "Dressing" of Operator Families	280
References	284
8. <i>Generalized Matrix Form of the Inverse Scattering Method</i> . By M. Wadati .	287
8.1 Historical Remarks	287
8.2 The Inverse Scattering Problem	288
8.2.1 Jost Functions	289
8.2.2 A Fundamental System of Solutions	290
8.2.3 Bound States	290
8.2.4 The Gelfand-Levitan-Marchenko Equation	291
8.3 The Inverse Scattering Method and Solvable Equations	292
8.3.1 The Korteweg-de Vries Equation in Matrix Form	293
8.3.2 The Nonlinear Schrödinger Equation in Matrix Form	294
8.4 Extension to Lattice Problems	295
8.4.1 Volterra Systems	295
8.4.2 The Toda Lattice Equation and a Nonlinear Self-Dual Network Equation	296
8.4.3 Discrete Nonlinear Schrödinger Equation	297
8.5 Concluding Remarks	298
References	299
9. <i>Nonlinear Evolution Equations Solvable by the Inverse Spectral Transform Associated with the Matrix Schrödinger Equation</i> By F. Calogero, and A. Degasperis (With 1 Figure)	301
9.1 Direct and Inverse Matrix Schrödinger Problem; Notation	301
9.2 Generalized Wronskian Relations; Basic Formulae	303
9.3 Nonlinear Evolution Equations Solvable by the Inverse Spectral Transform; Solitons	304
9.4 The Boomeron Equation and Other Solvable Nonlinear Evolution Equations Related to it; Boomerons	311

9.5	Bäcklund Transformations	319
9.6	Nonlinear Superposition Principle	321
9.7	Conserved Quantities	321
9.8	Generalized Resolvent Formula	322
9.9	Nonlinear Operator Identities	323
	References	323
10.	<i>A Method of Solving the Periodic Problem for the KdV Equation and Its Generalizations.</i> By S.P. Novikov	325
10.1	One-Dimensional Systems Admitting a Lax Representation; Their Stationary Solutions	325
10.2	Finite-Zoned Linear Operators	327
10.3	Hamiltonian Formalism for KdV in Stationary and Nonstationary Problems	331
10.4	The Akhiezer Function and Its Applications	334
	References	337
11.	<i>A Hamiltonian Interpretation of the Inverse Scattering Method.</i> By L.D. Faddeev	339
11.1	The Hamiltonian Formulation	339
11.2	Complete Integrability of the Nonlinear Schrödinger Equation	344
11.3	Applications to the Quantization Problem	350
	References	354
12.	<i>Quantum Solitons in Statistical Physics.</i> By A.H. Luther	355
12.1	Preliminary Remarks	355
12.2	Quantization and Quantum Solitons	357
12.3	Continuum Field Equations	361
12.4	Eigenvalue Spectrum	369
	References	371
	<i>Further Remarks on John Scott Russel and on the Early History of His Solitary Wave</i>	373
	<i>Note Added in Proof (Chapter 1)</i>	379
	<i>Additional References with Titles</i>	381
	<i>Subject Index</i>	383