Contents

1.	Goal		
	1.1	Order and Disorder: Some Typical Phenomena	1
	1.2	Some Typical Problems and Difficulties	12
	1.3	How We Shall Proceed	15
2.	Probability		
	2.1	Object of Our Investigations: The Sample Space	17
	2.2	Random Variables	19
	2.3	Probability	20
	2.4	Distribution	21
	2.5	Random Variables with Densities	24
	2.6	Joint Probability	26
	2.7	Mathematical Expectation $E(X)$, and Moments	28
	2.8	Conditional Probabilities	29
	2.9	Independent and Dependent Random Variables	30
	2.10	*Generating Functions and Characteristic Functions	31
	2.11	A Special Probability Distribution: Binomial Distribution	33
		The Poisson Distribution	36
	2.13	The Normal Distribution (Gaussian Distribution)	37
	2.14	Stirling's Formula	39
	2.15	*Central Limit Theorem	39
3.	Information		
	3.1	Some Basic Ideas	41
	3.2*	Information Gain: An Illustrative Derivation	46
	3.3	Information Entropy and Constraints	48
	3.4	An Example from Physics: Thermodynamics	53
	3.5*	An Approach to Irreversible Thermodynamics	57
	3.6	Entropy—Curse of Statistical Mechanics?	66
4.	Chance		
	4.1	A Model of Brownian Movement	69
	4.2	The Random Walk Model and Its Master Equation	75
	4.3*	Joint Probability and Paths. Markov Processes. The Chapman-	
		Kolmogorov Equation. Path Integrals	79

^{*} Sections with an asterisk in the heading may be omitted during a first reading.

X	Contents

	4.4*	How to Use Joint Probabilities. Moments. Characteristic	
		Function. Gaussian Processes	85
	4.5	The Master Equation	88
	4.6	Exact Stationary Solution of the Master Equation for Systems	
	4.7*	in Detailed Balance	
		Eigenvalues and Eigenstates	92
	4.8*	Kirchhoff's Method of Solution of the Master Equation	95
	4.9*	Theorems about Solutions of the Master Equation	97
	4.10	The Meaning of Random Processes. Stationary State, Fluctuations,	
	4.11	*Master Equation and Limitations of Irreversible Thermo-	98
		dynamics	102
5.	Nece	essity	
	5.1	Dynamic Processes	105
		Critical Points and Trajectories in a Phase Plane. Once Again	
	F 44	Limit Cycles	
		Stability	120
	5.4 5.5*	Examples and Exercises on Bifurcation and Stability	126
		to Thom's Theory of Catastrophes	133
6.	Char	nce and Necessity	
	6.1	Langevin Equations: An Example	147
		Reservoirs and Random Forces	152
	6.3	The Fokker-Planck Equation	
	6.4	Some Properties and Stationary Solutions of the Fokker-Planck	
		Equation	165
	6.5	Time-Dependent Solutions of the Fokker-Planck Equation	172
	6.6*	Solution of the Fokker-Planck Equation by Path Integrals	176
	6.7 6.8	Phase Transition Analogy	179
		Dependent Order Parameter	186
7.	Self-	Organization	
	7.1		101
	7.1		191
		Self-Organization	194
	7.3 7.4*	The Role of Fluctuations: Reliability or Adaptibility? Switching Adiabatic Elimination of Fast Relaxing Variables from the	200
		Fokker-Planck Equation	202
	7.5*	Adiabatic Elimination of Fast Relaxing Variables from the	
	7.6		204
	,	the Mathematical Approach	205

		Contents	ΧI
	7.7*	Generalized Ginzburg-Landau Equations for Nonequilibrium Phase Transitions	206
	7.8*	Higher-Order Contributions to Generalized Ginzburg-Landau	
	7.9*	Equations	
		Systems	
		*Soft-Mode Instability *Hard-Mode Instability	
8.	Phys	sical Systems	
	8.1	Cooperative Effects in the Laser: Self-Organization and Phase	225
	8.2	Transition	
	8.3	The Laser Equations in the Mode Picture	
	8.4	The Single-Mode Laser	
	8.5	The Multimode Laser	
	8.6	Laser with Continuously Many Modes. Analogy with Superconductivity	
	8.7	First-Order Phase Transitions of the Single-Mode Laser	
	8.8	Hierarchy of Laser Instabilities and Ultrashort Laser Pulses	
	8.9	Instabilities in Fluid Dynamics: The Bénard and Taylor Problems	245
	8 10	The Basic Equations	
		Damped and Neutral Solutions ($R \le R_c$)	
		Solution Near $R = R_c$ (Nonlinear Domain). Effective Langevin Equations	
	8 12	The Fokker-Planck Equation and Its Stationary Solution	
		A Model for the Statistical Dynamics of the Gunn Instability	
	0.15	Near Threshold	
	8.15	Elastic Stability: Outline of Some Basic Ideas	258
9.	Chei	nical and Biochemical Systems	
	9.1	Chemical and Biochemical Reactions	263
	9.2	Deterministic Processes, Without Diffusion, One Variable	263
	9.3	Reaction and Diffusion Equations	268
	9.4	Reaction-Diffusion Model with Two or Three Variables: The	
		Brusselator and the Oregonator	270
	9.5	Stochastic Model for a Chemical Reaction Without Diffusion.	
		Birth and Death Processes. One Variable	277
	9.6	Stochastic Model for a Chemical Reaction with Diffusion. One	202
	9.7*	Variable Stochastic Treatment of the Brusselator Close to Its Soft-Mode	
		Instability	
	9.8	Chemical Networks	290

.

XII Contents

10.	Applications to Biology	
	10.1 Ecology, Population-Dynamics	293
	10.2 Stochastic Models for a Predator-Prey System	
	10.3 A Simple Mathematical Model for Evolutionary Processes	298
	10.4 A Model for Morphogenesis	
	10.5 Order Parameters and Morphogenesis	
	10.6 Some Comments on Models of Morphogenesis	313
11.	Sociology: A Stochastic Model for the Formation of Public Opinion	315
12.	Chaos	
	12.1 What is Chaos?	319
	12.2 The Lorenz Model. Motivation and Realization	
	12.3 How Chaos Occurs	322
	12.4 Chaos and the Failure of the Slaving Principle	327
	12.5 Correlation Function and Frequency Distribution	329
	12.6 Further Examples of Chaotic Motion	331
13.	Some Historical Remarks and Outlook	333
Ref	ferences, Further Reading and Comments	337
Sul	biect Index	351