

Contents

Part I

1	Fundamental Concepts	3
1.1	Genesis and a Brief Introduction of Semirings	3
1.2	Basic Definitions, Some Examples, and Important Applications of Semirings	4
1.3	Preliminary Definitions and Related Concepts	6
1.4	Algebraic Preliminaries of Semirings	8
1.5	Fuzzy Sets, Basic Definitions, Examples and Their Applications	10
1.6	Fuzzy Algebraic Structures: A Brief Review	12
2	Fuzzy Ideals of Semirings	15
2.1	Sum and Product of Fuzzy Sets	15
2.2	Regular Semirings	23
2.3	Weakly Regular Semirings	23
2.4	Fully Idempotent Semirings	25
3	Fuzzy Subsemimodules over Semirings	31
3.1	Fuzzy Subsemimodules	31
3.2	Fuzzy Prime Subsemimodules of a Semimodule over a Semiring	41
3.3	Fully Fuzzy Prime Semirings	44
3.4	Fuzzy Prime Subsemimodules	46
4	Fuzzy k-Ideals of Semirings	53
4.1	k -Product and k -Sum of Fuzzy Subsets	55
4.2	k -Regular Semirings	63
4.3	Right k -Weakly Regular Semirings	63
4.4	Prime and Fuzzy Prime Right k -Ideals	66
4.5	Idempotent k -Ideals	69
4.6	Prime and Semiprime Fuzzy k -Ideals	71

4.7	k -Fuzzy Ideals and k -Semirings	75
4.8	Fuzzy Congruences	78
5	Fuzzy Quasi-ideals and Fuzzy Bi-ideals in Semirings	83
5.1	Fuzzy Quasi-ideals and Bi-ideals	83
5.2	Regular Semirings in Terms of Fuzzy Quasi-ideals and Fuzzy Bi-ideals	88
5.3	Intra-regular Semirings in Terms of Fuzzy Quasi-ideals and Fuzzy Bi-ideals	91
5.4	Fuzzy k -Bi-ideals and Fuzzy k -Quasi-ideals of Semirings	94
5.5	k -Regular Semirings	98
5.6	k -Intra-regular Semirings	101
6	$(\in, \in \vee q)$-Fuzzy Ideals in Semirings	105
6.1	(α, β) -Fuzzy Ideals	105
6.2	$(\in, \in \vee q)$ -Fuzzy Ideals	107
6.3	Regular Semirings	113
6.4	Intra-regular Semirings	116
6.5	$(\in, \in \vee q)$ -Fuzzy k -Ideals	118
6.6	k -Regular Semirings	121
6.7	k -Intra-regular Semirings	122
7	$(\overline{\in}, \overline{\in} \vee \overline{q})$-Fuzzy Ideals in Semirings	123
7.1	$(\overline{\in}, \overline{\in} \vee \overline{q})$ -Fuzzy Ideals	123
7.2	Regular Semirings	129
7.3	Intra-regular Semirings	132
7.4	$(\overline{\in}, \overline{\in} \vee \overline{q})$ -Fuzzy k -Ideals	134
7.5	k -Regular Semirings	137
7.6	k -Intra-regular Semirings	138
8	Fuzzy Ideals with Thresholds	141
8.1	Fuzzy Ideals with Threshold (α, β)	141
8.2	Regular Semirings	147
8.3	Intra-regular Semirings	151
8.4	Fuzzy k -Ideals with Thresholds (α, β)	153
8.5	k -Regular Semirings	157
8.6	k -Intra-regular Semirings	158
8.7	Ternary Semirings	159

Part II Invited Chapters

9	On Fuzzy LD-Bigroupoids	163
9.1	Preliminaries	163
9.2	LD -Bigroupoids	164
9.3	Fuzzy LD -Bigroupoids	165
9.4	Mean Value Property	166

9.5	Dot-Prime and Scalar Barrier Value	167
9.6	Null-Elements	168
9.7	Star-rising and Dot-rising	169
9.8	Specializations of fuzzy <i>LD</i> -Bigroupoids.	170
9.9	Duality	173
References		174
10	Semiring Parsing	175
10.1	Introduction	175
10.2	Context-Free Parsing Algorithms	176
10.2.1	Recognition and Parsing Algorithms for Context-Free Languages	176
10.2.2	Probabilistic Context-Free Grammars (PCFGs)	178
10.3	Deductive Parsing	181
10.3.1	Components of a Deductive System	182
10.3.2	Item-Based Descriptions	184
10.4	Semiring Parsing	185
10.4.1	Principles of Semiring Parsing	185
10.4.2	Examples: Inside Semiring and Viterbi Semiring	185
10.4.3	Some Classic Semirings in Semiring Parsing	187
10.4.4	Semiring Parsing – A Prototype Implementation	187
10.5	Summary	191
References		192
11	Coverings and Decompositions of Semiring-Weighted Finite Transition Systems	193
11.1	Introduction	193
11.2	Semirings and Weighted Finite Transition Systems	195
11.3	Coverings	196
11.4	Cascade Products	199
11.5	Admissible Partitions	203
11.6	A Decomposition by Cascade Products	206
11.7	What about Strong Coverings?	211
11.8	Conclusion	214
References		214
References		217
Index		225