

Contents

Preface	vii
Nomenclature	xi
1 Dynamical systems	1
1.1 Main definitions	1
1.2 Embedding of a discrete dynamical system into a flow	10
1.3 Local Poincaré diffeomorphism	11
1.4 Time-periodic systems of differential equations	14
1.5 Action of an Abelian group	15
2 Topologies on spaces of dynamical systems	16
2.1 C^0 -topology	16
2.2 C^1 -topology	17
2.3 Metrics on the space of systems of differential equations	18
2.4 Generic properties	24
2.5 Immersions and embeddings	24
3 Equivalence relations	26
3.1 Topological conjugacy	26
3.2 Topological equivalence of flows	30
3.3 Nonwandering set	30
3.4 Local equivalence	36
4 Hyperbolic fixed point	37
4.1 Hyperbolic linear mapping	37
4.2 The Grobman–Hartman theorem	40
4.3 Neighborhood of a hyperbolic fixed point	48
4.4 The stable manifold theorem	53
4.5 Hyperbolic periodic point	65
5 Hyperbolic rest point and hyperbolic closed trajectory	67
5.1 Hyperbolic rest point	67
5.2 Hyperbolic closed trajectory	72

6	Transversality	78
6.1	Transversality of mappings and submanifolds	78
6.2	Transversality condition	80
6.3	Palis lemma	82
6.4	Transversality and hyperbolicity for one-dimensional mappings	90
7	Hyperbolic sets	92
7.1	Definition of a hyperbolic set	92
7.2	Examples of hyperbolic sets	94
7.3	Basic properties of hyperbolic sets	97
7.4	Stable manifold theorem	101
7.5	Axiom A	103
7.6	Hyperbolic sets of flows	112
8	Anosov diffeomorphisms	119
9	Smale's horseshoe and chaos	126
9.1	Smale's horseshoe	126
9.2	Chaotic sets	131
9.3	Homoclinic points	132
10	Closing Lemma	135
11	C^0-generic properties of dynamical systems	141
11.1	Hausdorff metric	141
11.2	Semicontinuous mappings	142
11.3	Tolerance stability and Takens' theory	143
11.4	Attractors of dynamical systems	147
12	Shadowing of pseudotrajectories in dynamical systems	159
12.1	Definitions and results	159
12.2	Proof of Theorem 12.1	164
12.3	Proof of Theorem 12.2	172
12.4	Proof of Theorem 12.3	175
A	Scheme of the proof of the Mañé theorem	181
B	Lectures on the history of differential equations and dynamical systems	192
B.1	Differential equations and Newton's anagram	192
B.2	Development of the general theory	194

B.3	Linear equations and systems	198
B.4	Stability	203
B.5	Nonlocal qualitative theory. Dynamical systems	210
B.6	Structural stability	214
B.7	Dynamical systems with chaotic behavior	217
Bibliography		223
Index		227