

# Contents

## Part I Methodology: From Retarded to Neutral Continuous Delay Models

<b>Lyapunov Functionals and Matrices for Neutral Type Time Delay Systems</b> .....	3
Vladimir L. Kharitonov	
1 Introduction .....	3
2 Preliminaries .....	4
2.1 System Description .....	4
2.2 Exponential Stability .....	5
2.3 Problem Formulation .....	5
3 Lyapunov Matrices and Functionals .....	6
3.1 Existence and Uniqueness Issues .....	7
3.2 Computational Issue .....	8
3.3 Spectral Properties .....	8
4 Lyapunov Functionals: A New Form .....	9
5 Quadratic Bounds .....	11
6 Applications .....	12
6.1 Exponential Estimates .....	12
6.2 Quadratic Performance Index .....	13
6.3 Robustness Bounds .....	14
6.4 The $\mathcal{H}_2$ Norm of a Transfer Matrix .....	15
7 Comments .....	15
References .....	16
<b>On the Stability of Positive Difference Equations</b> .....	19
Michael Di Loreto and Jean Jacques Loiseau	
1 Introduction .....	19
2 Stability of Difference Equations .....	21
2.1 Exponential Stability .....	21
2.2 Zeros Location of Difference Equations .....	22
2.3 Stability and Robust Stability of Difference Equations .....	24
2.4 Examples .....	26

3	Main Results .....	28
3.1	Positive Difference Equations .....	28
3.2	Stability of Monovariate Positive Difference Equations .....	28
3.3	Stability of Multivariate Positive Difference Equations .....	29
3.4	Asymptotic Behavior of Positive Difference Equations .....	30
4	Conclusions and Final Remarks .....	32
	References .....	33

## **Positivity of Complete Quadratic Lyapunov-Krasovskii Functionals in Time-Delay Systems .....**

Keqin Gu, Yashun Zhang, and Matthew Peet

1	Introduction .....	35
2	Positive Operators .....	36
3	Positive Quadratic Integral Expressions .....	39
4	Stability of Coupled Differential-Difference Equations .....	41
5	SOS Formulation .....	44
6	Numerical Example and Observation .....	46
7	Conclusion .....	46
	References .....	46

## **On Retarded Nonlinear Time-Delay Systems That Generate Neutral Input-Output Equations .....**

Milena Angelova and Miroslav Halás

1	Introduction .....	49
2	Algebraic Setting .....	50
3	Input-Output Representation .....	51
3.1	State Elimination Algorithm .....	51
3.2	Neutral Input-Output Equations .....	53
4	Main Result .....	54
4.1	Input-Output Equations for Linear Delay Systems .....	54
4.2	Input-Output Equations for Nonlinear Delay Systems .....	56
5	Summary .....	57
6	Retarded Input-Output Representation .....	57
7	Conclusions, Discussion and Open Problems .....	58
	References .....	59

## **Computation of Imaginary Axis Eigenvalues and Critical Parameters for Neutral Time Delay Systems .....**

Gilberto Ochoa, Sabine Mondié, and Vladimir L. Kharitonov

1	Introduction .....	61
2	Preliminary Results .....	62
2.1	System Description .....	62

2.2	Lyapunov-Krasovskii Functionals .....	63
2.3	Properties of the Lyapunov Matrix .....	63
3	Computation of the Lyapunov Matrix .....	64
4	Frequency Domain Stability Analysis .....	66
5	Proposed Methodology .....	67
6	Examples .....	68
7	Conclusions .....	71
	References .....	71
<b>Set-Induced Stability Results for Delay Difference Equations .....</b>		<b>73</b>
Rob H. Gielen, Mircea Lazar, and Sorin Olaru		
1	Introduction .....	73
2	Preliminaries .....	74
2.1	Delay Difference Equations .....	75
3	$\mathcal{D}$ -Contractive Sets and $\mathcal{H}\mathcal{L}$ -Stability .....	76
4	Necessary Conditions for $\mathcal{D}$ -Contractive Sets .....	79
5	Illustrative Example .....	81
6	Conclusion .....	83
	References .....	83
<b>Part II Systems, Signals and Applications</b>		
<b>Temperature and Heat Flux Dependence/Independence for Heat Equations with Memory .....</b>		<b>87</b>
Sergei Avdonin and Luciano Pandolfi		
1	Introduction .....	87
1.1	Further Applications and References .....	88
2	The Interpretation of Dependence/Independence of Heat and Flux .....	89
3	Strict Dependence at Small Times .....	90
4	Independence at Large Times .....	90
4.1	Preliminary Transformations and Asymptotic Estimates .....	91
4.2	Riesz Bases and Riesz Sequences .....	92
4.3	Temperature and Flux Regularity .....	94
4.4	The Proof of Independence .....	98
	References .....	100
<b>Identifiability and Algebraic Identification of Time Delay Systems .....</b>		<b>103</b>
Lotfi Belkoura		
1	Introduction .....	103
1.1	The Distribution Framework .....	104
2	Identifiability Analysis .....	105
2.1	Sufficiently Rich Input .....	107
3	Algebraic Identification: The Structured Case .....	108
3.1	Structured Signals and Their Annihilation .....	108

3.2	Application to a Single Delay Identification . . . . .	108
3.3	Application to a Simultaneous Parameters and Delay Identification: Experimental Example . . . . .	110
4	Algebraic Identification: The Unstructured Case . . . . .	112
4.1	The Cross Convolution Approach . . . . .	112
4.2	Application to a Delay Identification . . . . .	113
4.3	Experimental Results (Continued) . . . . .	115
5	Conclusion . . . . .	116
	References . . . . .	116

**Stability Analysis for a Consensus System of a Group of Autonomous Agents with Time Delays . . . . . 119**

Rudy Cepeda-Gomez and Nejat Olgac

1	Introduction . . . . .	119
2	Problem Statement . . . . .	121
3	Stability Analysis . . . . .	123
3.1	First Factor of (9) . . . . .	123
3.2	Second Factor of (9) . . . . .	125
3.3	Complete Stability Picture . . . . .	126
4	Simulations . . . . .	127
5	Conclusions . . . . .	130
	Appendix . . . . .	131
	References . . . . .	133

**State Space for Time Varying Delay . . . . . 135**

Erik I. Verriest

1	Introduction . . . . .	135
2	State . . . . .	136
3	Functional Differential Equation . . . . .	137
4	Non-differential Functional Equation . . . . .	141
5	Reachability of Systems with Fixed Point Delay . . . . .	142
5.1	PBH Test for Delay Systems . . . . .	142
5.2	State Augmentation . . . . .	144
6	Conclusions . . . . .	145
	References . . . . .	146

**Delays. Propagation. Conservation Laws. . . . . 147**

Vladimir Rășvan

1	Introduction and Basics . . . . .	147
2	Lossless and Distortionless Propagation . . . . .	149
3	The Multi-wave Case. Application to the Circulating Fuel Nuclear Reactors . . . . .	152
4	A Control Problem . . . . .	153
5	Dynamics and Control for Systems of Conservation Laws . . . . .	156
6	Conclusions . . . . .	158
	References . . . . .	158

**Equations with Advanced Arguments in Stick Balancing Models . . . . .** 161  
Tamas Insperger, Richard Wohlfart, Janos Turi, and Gabor Stepan

1 Introduction . . . . . 161

2 Different Mechanical Models . . . . . 162

3 Analysis of the Different Models . . . . . 164

4 Results and Conclusions . . . . . 172

References . . . . . 172

**Optimal Control with Preview for Lateral Steering of a Passenger Car: Design and Test on a Driving Simulator . . . . .** 173  
Louay Saleh, Philippe Chevrel, and Jean-François Lafay

1 Introduction . . . . . 173

2 Problem Statement . . . . . 174

3 The H2/LQ-Preview Problem Solution . . . . . 175

4 Solution Analysis . . . . . 177

5 Application to Car Lateral Steering Control . . . . . 178

5.1 Simplified Lateral Model of the Vehicle . . . . . 178

5.2 Experimental Setup and Results . . . . . 180

6 Preview Horizon and Weighting Matrix Impact . . . . . 183

7 Conclusion . . . . . 184

References . . . . . 185

**Local Asymptotic Stability Conditions for the Positive Equilibrium of a System Modeling Cell Dynamics in Leukemia . . . . .** 187  
Hitay Özbay, Catherine Bonnet, Houda Benjelloun, and Jean Clairambault

1 Introduction . . . . . 187

2 Mathematical Model of Cell Dynamics in Leukemia . . . . . 188

3 Stability Analysis for the Positive Equilibrium . . . . . 190

3.1 Local Asymptotic Stability for  $\mu > 0$  . . . . . 191

3.2 Local Asymptotic Stability for  $\mu < 0$  . . . . . 191

4 Conclusions . . . . . 196

References . . . . . 196

**Part III Numerical Methods**

**Design of Fixed-Order Stabilizing and  $\mathcal{H}_2$  -  $\mathcal{H}_\infty$  Optimal Controllers: An Eigenvalue Optimization Approach . . . . .** 201  
Wim Michiels

1 Introduction . . . . . 201

2 Solving Analysis Problems . . . . . 202

2.1 Computation of Characteristic Roots and the Spectral Abscissa . . . . . 203

2.2 Computation of  $\mathcal{H}_\infty$  Norms . . . . . 205

2.3 Computation of  $\mathcal{H}_2$  Norms . . . . . 206

3 Solving Synthesis Problems . . . . . 208

3.1 Stabilization . . . . . 209

3.2  $\mathcal{H}_\infty$  and  $\mathcal{H}_2$  Optimization Problems . . . . . 210

4	Software and Applications .....	211
5	Concluding Remarks .....	214
	References .....	214
<b>Discretization of Solution Operators for Linear Time Invariant - Time Delay Systems in Hilbert Spaces .....</b>		
217		
Dimitri Breda, Stefano Maset and Rossana Vermiglio		
1	Introduction .....	217
2	Solution Operators and Notation .....	219
3	Projection and Collocation .....	220
3.1	Convergence Analysis .....	221
3.2	The Matrix Form .....	224
4	Conclusions .....	226
	Appendix .....	226
	References .....	227
<b>The Infinite Arnoldi Method and an Application to Time-Delay Systems with Distributed Delays .....</b>		
229		
Elias Jarlebring, Wim Michiels, and Karl Meerbergen		
1	Introduction .....	229
2	Operator Formulation .....	230
3	The Infinite Arnoldi Method .....	231
4	Adaption for Time-Delay Systems with Distributed Delays .....	233
4.1	Computing $y_0$ for Distributed Delays .....	233
4.2	Connection with the Fourier Cosine Transform .....	235
5	Examples .....	236
5.1	Example 1: A Rectangular Function .....	236
5.2	Example 2: A Gaussian Distribution .....	237
6	Conclusions .....	238
	References .....	238
<b>A New Method for Delay-Independent Stability of Time-Delayed Systems .....</b>		
241		
Ali Fuat Ergenc		
1	Introduction and the Problem Statement .....	241
2	Preliminaries .....	242
3	Main Results .....	244
4	Example Case Studies .....	247
5	Conclusions .....	250
	References .....	250
<b>A Hybrid Method for the Analysis of Non-uniformly Sampled Systems .....</b>		
253		
Laurentiu Hetel, Alexandre Kruszewski, and Jean-Pierre Richard		
1	Introduction .....	253
2	Problem Description .....	255

3	Lower Estimate of the Lyapunov Exponent .....	256
3.1	Time-Delay Model of the System .....	256
3.2	Theoretical Evaluation .....	257
3.3	Numerical Evaluation .....	258
3.4	Numerical Improvements .....	260
4	Numerical Example .....	262
5	Conclusion .....	262
	References .....	263

**A Numerical Method for the Construction of Lyapunov Matrices for Linear Periodic Systems with Time Delay .....** 265

Olga N. Letyagina and Alexey P. Zhabko

1	Introduction .....	265
2	Preliminaries .....	266
3	Complete Type Lyapunov Functionals .....	267
4	Existence Issue .....	269
5	Exponential Estimation and Robust Stability .....	272
5.1	Exponential Estimation .....	272
5.2	Robust Stability .....	272
6	Computation Issue .....	274
	References .....	275

**Polytopic Discrete-Time Models for Systems with Time-Varying Delays .....** 277

Warody Lombardi, Sorin Olaru, and Silviu-Iulian Niculescu

1	Introduction .....	277
2	Problem Formulation .....	278
3	Polytopic Embeddings .....	279
3.1	Non-defective System Matrix with Real Eigenvalues ...	280
3.2	Defective System Matrix with Real Eigenvalues .....	281
3.3	Non-defective System Matrix with Complex-Conjugated Eigenvalues .....	283
4	Towards a Less Conservative Simplex Embedding .....	283
5	Conclusion .....	287
	References .....	287

**Part IV Predictor-Based Control and Compensation**

**Predictor Feedback: Time-Varying, Adaptive, and Nonlinear .....** 291

Miroslav Krstic

1	Introduction .....	291
2	Lyapunov Functional and Its Immediate Benefits .....	292
3	Delay-Robustness, Delay-Adaptivity, and Time-Varying Delays .....	294
3.1	Robustness to Delay Mismatch .....	294
3.2	Delay-Adaptive Predictor .....	295

3.3	Time-Varying Input Delay .....	297
4	Predictor Feedback for Nonlinear Systems .....	299
5	Non-holonomic Unicycle Controlled over a Network .....	302
6	State-Dependent Delay .....	303
7	Conclusions .....	305
	References .....	305
<b>Smoothing Techniques-Based Distributed Model Predictive Control</b>		
<b>Algorithms for Networks .....</b>		<b>307</b>
Ion Necoara, Ioan Dumitrache, and Johan A.K. Suykens		
1	Introduction .....	307
2	Application of Smoothing Techniques to Separable Convex Problems .....	308
2.1	Proximal Center Decomposition Method .....	310
2.2	Interior-Point Lagrangian Decomposition Method .....	311
2.3	Application of Smoothing Techniques to Separable Non-convex Problems .....	312
3	Distributed Model Predictive Control .....	313
3.1	Distributed MPC for Coupling Non-linear Dynamics ...	313
3.2	Distributed MPC for Coupling Linear Dynamics .....	315
3.3	Practical Implementation .....	316
4	Conclusions .....	317
	References .....	318
<b>Model Predictive Control with Delay Compensation for Air-to-Fuel</b>		
<b>Ratio Control .....</b>		<b>319</b>
Sergio Trimboli, Stefano Di Cairano, Alberto Bemporad, and Ilya V. Kolmanovsky		
1	Introduction .....	320
2	Model of AFR and Oxygen Storage .....	322
2.1	Air-to-Fuel Ratio Dynamics .....	322
2.2	Oxygen Storage Dynamics .....	323
3	Model Predictive Control with Delay Compensation .....	324
3.1	State Predictor .....	324
3.2	Model Predictive Controller .....	325
4	Simulation Results .....	327
5	Conclusions .....	330
	References .....	330
<b>Observer-Based Stabilizing Control for a Class of Nonlinear Retarded</b>		
<b>Systems .....</b>		<b>331</b>
Alfredo Germani, Costanzo Manes, and Pierdomenico Pepe		
1	Introduction .....	331
2	Preliminaries .....	332
3	Separation Theorem: Global Results .....	333
4	Separation Theorem: Local Results .....	336



5	Numerical Example .....	339
6	Conclusions .....	340
	References .....	341
<b>Cascade Control for Time Delay Plants .....</b>		<b>343</b>
Pavel Zítek, Vladimír Kučera, and Tomáš Vyhliďal		
1	Introduction .....	343
2	Meromorphic Quasi-integration Transfer Function .....	345
3	Parameterization Based Design of the Secondary Control Loop .....	347
4	Master Controller Design .....	350
5	Concluding Remarks .....	354
	References .....	354
<b>Design of Terminal Cost Functionals and Terminal Regions for Model Predictive Control of Nonlinear Time-Delay Systems .....</b>		<b>355</b>
Marcus Reble and Frank Allgöwer		
1	Introduction .....	355
2	Problem Setup .....	357
3	Model Predictive Control for Nonlinear Time-Delay Systems ...	358
4	General Linearization-Based Design .....	360
5	Design by Combination of Lyapunov-Krasovskii and Lyapunov-Razumikhin Arguments .....	362
6	Brief Discussion of Both Results .....	364
7	Conclusions .....	365
	References .....	365
<b>Part V Networked Control Systems and Multi-agent Systems</b>		
<b>Networked Control under Time-Synchronization Errors .....</b>		<b>369</b>
Alexandre Seuret and Karl H. Johansson		
1	Introduction .....	369
2	Preliminaries .....	370
	2.1 Definition of the Plant .....	371
	2.2 Synchronization and Delays Models .....	371
3	Observer-Based Networked Control .....	372
4	Stabilization under Synchronization Error .....	373
	4.1 Closed-Loop System .....	373
	4.2 Stability Criteria .....	374
5	Application to a Mobile Robot .....	377
6	Concluding Remarks .....	377
7	Proof of Theorem 1 .....	378
	References .....	381

<b>Modelling and Predictive Congestion Control of TCP Protocols</b> .....	383
Rafael C. Melo, Jean-Marie Farines, and Julio E. Normey-Rico	
1     Introduction .....	383
2     Modeling TCP Connections .....	385
2.1     Static Model .....	385
2.2     Dynamical Model .....	386
2.3     Simple Model for Control Purposes .....	387
2.4     Comparative Dynamical Models Simulation Results ...	388
3     Congestion Control .....	389
3.1     Case Study 1: Controller Performance Comparison ...	390
3.2     Case Study 2: Sample Time Effects .....	390
3.3     Case Study 3: RTT Variation Effects .....	391
4     Conclusion .....	392
References .....	393
 <b>Dependence of Delay Margin on Network Topology: Single</b>	
<b>Delay Case</b> .....	395
Wei Qiao and Rifat Sipahi	
1     Introduction .....	395
2     Preliminaries .....	397
2.1     Linear-Time Invariant Model with Delays .....	397
2.2     Stability Analysis .....	397
3     Numerical Results .....	398
3.1     Dependence of Delay Margin on Topology - Weak	
Damping Case .....	398
3.2     Effects of Topology Transition on Stability .....	400
3.3     Stability Analysis with Respect to Damping .....	401
4     Conclusion .....	403
References .....	403
 <b>Consensus in Networks under Transmission Delays and the</b>	
<b>Normalized Laplacian</b> .....	407
Fatihcan M. Atay	
1     Introduction .....	407
2     The Normalized Laplacian and Consensus in the Absence of	
Delays .....	409
3     Consensus under Transmission Delays .....	410
4     Remarks .....	412
4.1     Zero Eigenvalue and Spanning Trees .....	412
4.2     Undirected Networks .....	413
4.3     Normalized versus Non-normalized Laplacian .....	413
4.4     Transmission versus Processing Delays .....	414
4.5     Distributed Delays, Discrete-Time Systems .....	415
References .....	415

**Consensus with Constrained Convergence Rate and Time-Delays . . . . . 417**  
Irinel-Constantin Morărescu, Silviu-Iulian Niculescu, and Antoine Girard

1 Introduction . . . . . 417

2 Preliminaries . . . . . 418

2.1 Algebraic Graph Theory Elements . . . . . 418

2.2 Consensus Protocol . . . . . 419

3 Consensus Problem for Networks with Fixed Topology . . . . . 421

3.1 Convergence Result . . . . . 421

3.2 Delay Margin Assuring a Fixed Network Topology . . . . . 423

4 Agreement Speed in Networks with Dynamic Topology . . . . . 425

5 Numerical Examples . . . . . 427

6 Conclusions . . . . . 428

References . . . . . 428

**$H_\infty$  Control of Networked Control Systems via Discontinuous  
Lyapunov Functionals . . . . . 429**  
Kun Liu and Emilia Fridman

1 Introduction . . . . . 429

2 Problem Formulation . . . . . 431

3 Main Results . . . . . 433

3.1 Exponential Stability and  $L_2$ -Gain Analysis . . . . . 433

3.2 Application to Network-Based  
Static Output-Feedback Design . . . . . 436

4 Examples . . . . . 437

5 Conclusions . . . . . 439

References . . . . . 439

**Index . . . . . 441**