

Contents

Part I BASIC DEFINITIONS

- 1. General Introduction 3**
 - Limit Points 5
 - Closures and Interiors 6
 - Countability Properties 7
 - Functions 7
 - Filters 9
- 2. Separation Axioms 11**
 - Regular and Normal Spaces 12
 - Completely Hausdorff Spaces 13
 - Completely Regular Spaces 13
 - Functions, Products, and Subspaces 14
 - Additional Separation Properties 16
- 3. Compactness 18**
 - Global Compactness Properties 18
 - Localized Compactness Properties 20
 - Countability Axioms and Separability 21
 - Paracompactness 22
 - Compactness Properties and T_1 Axioms 24
 - Invariance Properties 26
- 4. Connectedness 28**
 - Functions and Products 31
 - Disconnectedness 31
 - Biconnectedness and Continua 33

5. Metric Spaces	34
Complete Metric Spaces	36
Metrizability	37
Uniformities	37
Metric Uniformities	38

Part II COUNTEREXAMPLES

1. Finite Discrete Topology	41
2. Countable Discrete Topology	41
3. Uncountable Discrete Topology	41
4. Indiscrete Topology	42
5. Partition Topology	43
6. Odd-Even Topology	43
7. Deleted Integer Topology	43
8. Finite Particular Point Topology	44
9. Countable Particular Point Topology	44
10. Uncountable Particular Point Topology	44
11. Sierpinski Space	44
12. Closed Extension Topology	44
13. Finite Excluded Point Topology	47
14. Countable Excluded Point Topology	47
15. Uncountable Excluded Point Topology	47
16. Open Extension Topology	47
17. Either-Or Topology	48
18. Finite Complement Topology on a Countable Space	49
19. Finite Complement Topology on an Uncountable Space	49
20. Countable Complement Topology	50
21. Double Pointed Countable Complement Topology	50
22. Compact Complement Topology	51
23. Countable Fort Space	52
24. Uncountable Fort Space	52
25. Fortissimo Space	53
26. Arens-Fort Space	54
27. Modified Fort Space	55
28. Euclidean Topology	56
29. The Cantor Set	57
30. The Rational Numbers	59
31. The Irrational Numbers	59
32. Special Subsets of the Real Line	60
33. Special Subsets of the Plane	61
34. One Point Compactification Topology	63

35. One Point Compactification of the Rationals	63
36. Hilbert Space	64
37. Fréchet Space	64
38. Hilbert Cube	65
39. Order Topology	66
40. Open Ordinal Space $[0, \Gamma)$ ($\Gamma < \Omega$)	68
41. Closed Ordinal Space $[0, \Gamma]$ ($\Gamma < \Omega$)	68
42. Open Ordinal Space $[0, \Omega)$	68
43. Closed Ordinal Space $[0, \Omega]$	68
44. Uncountable Discrete Ordinal Space	70
45. The Long Line	71
46. The Extended Long Line	71
47. An Altered Long Line	72
48. Lexicographic Ordering on the Unit Square	73
49. Right Order Topology	74
50. Right Order Topology on R	74
51. Right Half-Open Interval Topology	75
52. Nested Interval Topology	76
53. Overlapping Interval Topology	77
54. Interlocking Interval Topology	77
55. Hjalmar Ekdal Topology	78
56. Prime Ideal Topology	79
57. Divisor Topology	79
58. Evenly Spaced Integer Topology	80
59. The p -adic Topology on Z	81
60. Relatively Prime Integer Topology	82
61. Prime Integer Topology	82
62. Double Pointed Reals	84
63. Countable Complement Extension Topology	85
64. Smirnov's Deleted Sequence Topology	86
65. Rational Sequence Topology	87
66. Indiscrete Rational Extension of R	88
67. Indiscrete Irrational Extension of R	88
68. Pointed Rational Extension of R	88
69. Pointed Irrational Extension of R	88
70. Discrete Rational Extension of R	90
71. Discrete Irrational Extension of R	90
72. Rational Extension in the Plane	91
73. Telophase Topology	92
74. Double Origin Topology	92
75. Irrational Slope Topology	93
76. Deleted Diameter Topology	94

77. Deleted Radius Topology	94
78. Half-Disc Topology	96
79. Irregular Lattice Topology	97
80. Arens Square	98
81. Simplified Arens Square	100
82. Niemytzki's Tangent Disc Topology	100
83. Metrizable Tangent Disc Topology	103
84. Sorgenfrey's Half-Open Square Topology	103
85. Michael's Product Topology	105
86. Tychonoff Plank	106
87. Deleted Tychonoff Plank	106
88. Alexandroff Plank	107
89. Dieudonne Plank	108
90. Tychonoff Corkscrew	109
91. Deleted Tychonoff Corkscrew	109
92. Hewitt's Condensed Corkscrew	111
93. Thomas' Plank	113
94. Thomas' Corkscrew	113
95. Weak Parallel Line Topology	114
96. Strong Parallel Line Topology	114
97. Concentric Circles	116
98. Appert Space	117
99. Maximal Compact Topology	118
100. Minimal Hausdorff Topology	119
101. Alexandroff Square	120
102. Z^Z	121
103. Uncountable Products of Z^+	123
104. Baire Product Metric on R^ω	124
105. I^I	125
106. $[0, \Omega) \times I^I$	126
107. Helly Space	127
108. $C[0, 1]$	128
109. Box Product Topology on R^ω	128
110. Stone-Čech Compactification	129
111. Stone-Čech Compactification of the Integers	132
112. Novak Space	134
113. Strong Ultrafilter Topology	135
114. Single Ultrafilter Topology	136
115. Nested Rectangles	137
116. Topologist's Sine Curve	137
117. Closed Topologist's Sine Curve	137
118. Extended Topologist's Sine Curve	137
119. The Infinite Broom	139

120. The Closed Infinite Broom	139
121. The Integer Broom	140
122. Nested Angles	140
123. The Infinite Cage	141
124. Bernstein's Connected Sets	142
125. Gustin's Sequence Space	142
126. Roy's Lattice Space	143
127. Roy's Lattice Subspace	143
128. Cantor's Leaky Tent	145
129. Cantor's Teepee	145
130. A Pseudo-Arc	147
131. Miller's Biconnected Set	148
132. Wheel without Its Hub	150
133. Tangora's Connected Space	150
134. Bounded Metrics	151
135. Sierpinski's Metric Space	152
136. Duncan's Space	153
137. Cauchy Completion	154
138. Hausdorff's Metric Topology	154
139. The Post Office Metric	155
140. The Radial Metric	155
141. Radial Interval Topology	156
142. Bing's Discrete Extension Space	157
143. Michael's Closed Subspace	157

Part III METRIZATION THEORY

Conjectures and Counterexamples	161
---------------------------------	-----

Part IV APPENDICES

Special Reference Charts	185
Separation Axiom Chart	187
Compactness Chart	188
Paracompactness Chart	190
Connectedness Chart	191
Disconnectedness Chart	192
Metrizability Chart	193
General Reference Chart	195
Problems	205
Notes	213
Bibliography	228

<i>Index</i>	236
--------------	-----