

# Contents

|                                                                  |     |
|------------------------------------------------------------------|-----|
| <b>Introduction</b> .....                                        | 1   |
| 1. This is a book... ..                                          | 1   |
| 2. How to use this book .....                                    | 2   |
| 3. About the English edition .....                               | 3   |
| 4. Acknowledgements .....                                        | 3   |
| <b>I. Affine Geometry</b> .....                                  | 7   |
| 1. Affine spaces .....                                           | 7   |
| 2. Affine mappings .....                                         | 14  |
| 3. Using affine mappings: three theorems in plane geometry ..... | 23  |
| 4. Appendix: a few words on barycenters .....                    | 26  |
| 5. Appendix: the notion of convexity .....                       | 28  |
| 6. Appendix: Cartesian coordinates in affine geometry .....      | 30  |
| Exercises and problems .....                                     | 32  |
| <b>II. Euclidean Geometry, Generalities</b> .....                | 43  |
| 1. Euclidean vector spaces, Euclidean affine spaces .....        | 43  |
| 2. The structure of isometries .....                             | 46  |
| 3. The group of linear isometries .....                          | 52  |
| Exercises and problems .....                                     | 58  |
| <b>III. Euclidean Geometry in the Plane</b> .....                | 65  |
| 1. Angles .....                                                  | 65  |
| 2. Isometries and rigid motions in the plane .....               | 76  |
| 3. Plane similarities .....                                      | 79  |
| 4. Inversions and pencils of circles .....                       | 83  |
| Exercises and problems .....                                     | 98  |
| <b>IV. Euclidean Geometry in Space</b> .....                     | 113 |
| 1. Isometries and rigid motions in space .....                   | 113 |
| 2. The vector product, with area computations .....              | 116 |
| 3. Spheres, spherical triangles .....                            | 120 |
| 4. Polyhedra, Euler formula .....                                | 122 |
| 5. Regular polyhedra .....                                       | 126 |
| Exercises and problems .....                                     | 130 |
| <b>V. Projective Geometry</b> .....                              | 143 |
| 1. Projective spaces .....                                       | 143 |
| 2. Projective subspaces .....                                    | 145 |

|                                                                      |            |
|----------------------------------------------------------------------|------------|
| 3. Affine <i>vs</i> projective .....                                 | 147        |
| 4. Projective duality .....                                          | 153        |
| 5. Projective transformations .....                                  | 155        |
| 6. The cross-ratio .....                                             | 161        |
| 7. The complex projective line and the circular group .....          | 164        |
| Exercises and problems .....                                         | 170        |
| <b>VI. Conics and Quadrics .....</b>                                 | <b>183</b> |
| 1. Affine quadrics and conics, generalities .....                    | 184        |
| 2. Classification and properties of affine conics .....              | 189        |
| 3. Projective quadrics and conics .....                              | 200        |
| 4. The cross-ratio of four points on a conic and Pascal's theorem .. | 208        |
| 5. Affine quadrics, <i>via</i> projective geometry .....             | 210        |
| 6. Euclidean conics, <i>via</i> projective geometry .....            | 215        |
| 7. Circles, inversions, pencils of circles .....                     | 219        |
| 8. Appendix: a summary of quadratic forms .....                      | 225        |
| Exercises and problems .....                                         | 233        |
| <b>VII. Curves, Envelopes, Evolutes .....</b>                        | <b>247</b> |
| 1. The envelope of a family of lines in the plane .....              | 248        |
| 2. The curvature of a plane curve .....                              | 254        |
| 3. Evolutes .....                                                    | 256        |
| 4. Appendix: a few words on parametrized curves .....                | 258        |
| Exercises and problems .....                                         | 261        |
| <b>VIII. Surfaces in 3-dimensional Space .....</b>                   | <b>269</b> |
| 1. Examples of surfaces in 3-dimensional space .....                 | 269        |
| 2. Differential geometry of surfaces in space .....                  | 271        |
| 3. Metric properties of surfaces in the Euclidean space .....        | 284        |
| 4. Appendix: a few formulas .....                                    | 294        |
| Exercises and problems .....                                         | 296        |
| <b>A few Hints and Solutions to Exercises .....</b>                  | <b>301</b> |
| Chapter I .....                                                      | 301        |
| Chapter II .....                                                     | 304        |
| Chapter III .....                                                    | 306        |
| Chapter IV .....                                                     | 314        |
| Chapter V .....                                                      | 321        |
| Chapter VI .....                                                     | 326        |
| Chapter VII .....                                                    | 332        |
| Chapter VIII .....                                                   | 336        |
| <b>Bibliography .....</b>                                            | <b>343</b> |
| <b>Index .....</b>                                                   | <b>347</b> |