

# Contents

|          |                                                 |           |
|----------|-------------------------------------------------|-----------|
| <b>1</b> | <b>Measurement Uncertainty</b>                  | <b>1</b>  |
| 1.1      | Historical review . . . . .                     | 3         |
| 1.2      | Goal of this Work . . . . .                     | 5         |
| 1.3      | What's new . . . . .                            | 6         |
| <b>2</b> | <b>Mathematical Concepts</b>                    | <b>9</b>  |
| 2.1      | What is a Measurement . . . . .                 | 9         |
| 2.1.1    | Definitions . . . . .                           | 9         |
| 2.2      | Different Approaches . . . . .                  | 13        |
| 2.2.1    | Random variables . . . . .                      | 13        |
| 2.2.2    | Mathematical Model . . . . .                    | 22        |
| 2.2.3    | ISO Guide: First Order Approximation . . . . .  | 27        |
| 2.2.4    | Simple Rules . . . . .                          | 27        |
| 2.2.5    | Monte Carlo Simulation . . . . .                | 28        |
| 2.3      | Calibration . . . . .                           | 30        |
| 2.3.1    | Calibration and Measurement . . . . .           | 30        |
| 2.3.2    | Calibration Models . . . . .                    | 31        |
| 2.3.3    | Computations . . . . .                          | 36        |
| 2.3.4    | Measurement . . . . .                           | 43        |
| 2.3.5    | Truncated SVD Solver . . . . .                  | 44        |
| 2.3.6    | Numerical Comparisons and Simulations . . . . . | 48        |
| 2.3.7    | Final remarks on Regression . . . . .           | 54        |
| <b>3</b> | <b>M description</b>                            | <b>57</b> |
| 3.1      | Compiling and Interpreting . . . . .            | 58        |
| 3.2      | Language M . . . . .                            | 59        |

|          |                                              |            |
|----------|----------------------------------------------|------------|
| 3.2.1    | Extended Backus-Naur Form (EBNF)             | 59         |
| 3.2.2    | Grammar of M                                 | 61         |
| 3.2.3    | Token definitions for M                      | 63         |
| 3.2.4    | Example: Grammar for Units                   | 63         |
| 3.2.5    | Complete Grammar of M                        | 65         |
| <b>4</b> | <b>Semantics of M</b>                        | <b>69</b>  |
| 4.1      | Expression                                   | 69         |
| 4.1.1    | Equation                                     | 70         |
| 4.1.2    | Leaf                                         | 70         |
| 4.1.3    | Units                                        | 72         |
| 4.2      | Attribute List                               | 74         |
| 4.3      | Functions                                    | 75         |
| 4.3.1    | Predefined Functions                         | 78         |
| 4.4      | Measurement                                  | 85         |
| <b>5</b> | <b>Implementation</b>                        | <b>93</b>  |
| 5.1      | General Architecture                         | 93         |
| 5.2      | COM Interface                                | 95         |
| 5.3      | Several Modules                              | 99         |
| 5.3.1    | SI-Units                                     | 99         |
| 5.3.2    | Parser                                       | 113        |
| 5.3.3    | Visitor Pattern                              | 116        |
| 5.3.4    | Drawing Cause and Effect Diagrams            | 122        |
| 5.3.5    | The program <i>Uncertainty Manager</i>       | 123        |
| <b>6</b> | <b>Real Life Application</b>                 | <b>127</b> |
| 6.1      | Certification of Bismuth content in SRM 3106 | 129        |
| 6.1.1    | NIST SRM Program                             | 129        |
| 6.1.2    | High Performance ICP-OES                     | 130        |
| 6.2      | Measuring calcium content in spring water    | 136        |
| <b>7</b> | <b>Conclusion</b>                            | <b>169</b> |