Contents

1 **Introduction to Lie groups and their representations** 1
1.1 Basic definitions 1
1.2 Lie groups and Lie algebras 6
1.2.1 Linear Lie groups 9
1.2.2 Real Lie algebras 12
1.3 Semi-simple Lie algebras and their representations 16
1.3.1 Classification of real semi-simple Lie algebras 17
1.3.2 Representations of semi-simple Lie algebras and linear Lie groups 22

2 **The rotation group** 27
2.1 Basic properties 27
2.2 Infinitesimal transformations and Lie algebras of the rotation group 32
2.3 Irreducible representations of $SO(3)$ and $SU(2)$ 33
2.4 Matrix representations of the rotation operators 37
2.5 Addition of angular momenta and Clebsch-Gordan coefficients 39
Problems 41

3 **The homogeneous Lorentz group** 43
3.1 Basic properties 43
3.2 The proper orthochronous Lorentz group $\mathcal{L}_+^\uparrow$ 46
3.3 Lie algebra of the group $\mathcal{L}_+^\uparrow$ 51
3.4 Irreducible representations of the group $\mathcal{L}_+^\uparrow$ 54
3.5 Irreducible representations of the complete Lorentz group 57
Problems 60

4 **The Poincaré transformations** 61
4.1 Group properties 61
4.2 Unitary representations of the proper orthochronous Poincaré group 64

Bibliografische Informationen
http://d-nb.info/1005223793

digitalisiert durch DEUTSCHE NATIONAL BIBLIOTHEK

Problems 70

5 One particle and two particle states 71
5.1 The little group 71
5.2 States of a massive particle 73
5.3 States of a massless particle 76
5.4 States of two particles 79
5.5 The ℓ-s coupling scheme 81
Problems 82

6 Discrete operations 83
6.1 Space inversion 83
6.2 Parity invariance 88
6.3 Time reversal 90
Problems 94

7 Relativistic equations 97
7.1 The Klein-Gordon equation 97
7.2 Extension to higher integer spins 100
7.3 The Maxwell equations 101
7.4 The Dirac equation 104
7.5 The Dirac equation for massless particles 108
7.6 Extension to higher half-integer spins 110
Problems 112

8 Unitary symmetries 113
8.1 Introduction 113
8.2 Generalities on symmetries of elementary particles 115
8.3 $U(1)$ invariance and Additive Quantum Numbers 117
8.4 Isospin invariance 120
8.4.1 Preliminary considerations 121
8.4.2 Isospin classification of hadrons 124
8.5 $SU(3)$ invariance 128
8.5.1 From $SU(2)$ to $SU(3)$ 129
8.5.2 Irreducible representations of $SU(3)$ 131
8.5.3 Lie algebra of $SU(3)$ 135
8.5.4 $SU(3)$ classification of hadrons 137
8.5.5 I-spin, U-spin and V-spin 146
8.5.6 The use of $SU(3)$ as exact symmetry 149
8.5.7 The use of $SU(3)$ as broken symmetry 153
8.6 Beyond $SU(3)$ 157
8.6.1 From flavor $SU(3)$ to color $SU(3)$ 158
8.6.2 The combination of internal symmetries with ordinary spin 159
8.6.3 Extensions of flavor $SU(3)$ 162

Problems ... 167

9 Gauge symmetries ... 169
9.1 Introduction ... 169
9.2 Invariance under group transformations and conservation laws 171
9.3 The gauge group $U(1)$ and Quantum Electrodynamics ... 175
9.4 The gauge group $SU(3)$ and Quantum Chromodynamics ... 176
9.5 The mechanism of spontaneous symmetry breaking ... 181
9.5.1 Spontaneous symmetry breaking of a discrete symmetry 181
9.5.2 Spontaneous symmetry breaking of a continuous global symmetry ... 184
9.5.3 Spontaneous symmetry breaking of a gauge symmetry: the Higgs mechanism ... 187
9.6 Spontaneous breaking of the chiral symmetry of QCD ... 189
9.7 The group $SU(2) \otimes U(1)$ and the electroweak interactions ... 192
9.7.1 Toward the unification of weak and electromagnetic interactions ... 192
9.7.2 Properties of the gauge bosons ... 196
9.7.3 The fermion sector of the Standard Model ... 199
9.8 Groups of Grand Unification ... 206
Problems ... 211

A Rotation matrices and Clebsch-Gordan coefficients ... 213
A.1 Reduced rotation matrices and spherical harmonics ... 213
A.2 Clebsch-Gordan coefficients ... 215

B Symmetric group and identical particles ... 219
B.1 Identical particles ... 219
B.2 Symmetric group and Young tableaux ... 220

C Young tableaux and irreducible representations of the unitary groups ... 225
C.1 Irreducible tensors with respect to $U(n)$ ... 225
C.2 Irreducible tensors with respect to $SU(n)$ ... 228
C.3 Reduction of products of irreducible representations ... 232
C.4 Decomposition of the IR's of $SU(n)$ with respect to given subgroups ... 233

Solutions ... 237

Bibliography ... 281

Index ... 287