

Contents

1	The BCS–BEC Crossover and the Unitary Fermi Gas	1
1.1	Introduction	1
1.2	Feshbach Resonance	6
1.3	Universality and Scale Invariance	10
1.4	Thermodynamics and Critical Temperature	11
1.5	Universal Tan Relations	14
1.6	The Unitary Fermi Gas as a Perfect Fluid?	16
1.7	RF Spectroscopy: Pairing Gap and Pseudogap	18
1.8	Spin Imbalance and the Fermi Polaron	23
1.9	FFLO Phases and Outlook	27
	References	28
2	The BEC–BCS Crossover: Some History and Some General Observations	33
2.1	Introduction	33
2.2	Some Prehistory	34
2.3	Some General Remarks on the Crossover Problem	38
	References	46
3	Crossovers in Unitary Fermi Systems	49
3.1	Introduction and the Bertsch Problem	49
3.2	Universality and Crossover	51
3.2.1	Universality in the Unitarity Limit	52
3.2.2	Thermodynamic Crossover Functions	54
3.2.3	Finite Temperature	56
3.2.4	Jastrow-Slater Approximation	57
3.2.5	Short Range Correlations	61
3.2.6	Instability in Multicomponent Systems	62
3.2.7	Repulsive Interactions and Itinerant Ferromagnetism	64

3.3	Pairing in Uniform Systems and the BCS–BEC Crossover . . .	66
3.3.1	BCS Limit	66
3.3.2	Induced Interactions	67
3.3.3	Leggett’s BCS–BEC Crossover	68
3.3.4	NSR	69
3.4	Atomic Traps and Nuclei	72
3.4.1	Scaling in Atomic Traps	73
3.4.2	Collective Modes	74
3.4.3	Shell Structure and Pairing in Atomic Traps	74
3.4.4	Pairing in Nuclei	75
3.4.5	Quark and Gluon Matter	78
3.5	Optical Lattices	79
3.5.1	On-Site Attraction and Pairing	80
3.5.2	On-Site Repulsion	84
3.5.3	High Temperature Superconductivity in Optical Lattices	89
3.6	Summary and Outlook	92
	References	94
4	Pairing Fluctuations Approach to the BCS–BEC Crossover	99
4.1	Introduction	99
4.2	Inclusion of Pairing Fluctuations	104
4.3	Bogoliubov and Popov Approaches, and the Boson–Boson Residual Interaction	109
4.4	Results for Thermodynamic and Dynamical Quantities	117
4.4.1	Thermodynamic Properties	117
4.4.2	Dynamical Properties	121
4.5	Concluding Remarks	124
	References	125
5	The Unitary Gas and its Symmetry Properties	127
5.1	Simple Facts About the Unitary Gas	128
5.1.1	What is the Unitary Gas?	128
5.1.2	Some Simple Properties of the Unitary Gas	131
5.1.3	Application: Inequalities on ξ and Finite-Temperature Quantities	134
5.1.4	Is the Unitary Gas Attractive or Repulsive?	136
5.1.5	Other Partial Waves, Other Dimensions	138
5.2	Various Models and General Relations	140
5.2.1	Lattice Models and General Relations	141
5.2.2	Zero-Range Model, Scale Invariance and Virial Theorem	149
5.2.3	Two-Channel Model and Closed-Channel Fraction . . .	153

5.3	Dynamical Symmetry of the Unitary Gas	161
5.3.1	Scaling Solution in a Time-Dependent Trap	161
5.3.2	SO(2,1) Dynamical Symmetry and the Decoupled Breathing Mode	165
5.3.3	Separability in Internal Hyperspherical Coordinates	170
5.3.4	Physical Consequences of the Separability.	174
5.3.5	Vanishing Bulk Viscosity	177
5.3.6	Short-Range Scaling Laws.	178
	Appendix 1: Effective Range in a Lattice Model	179
	Appendix 2: What is the Domain of a Hamiltonian?	181
	Appendix 3: Separability and Jacobi Coordinates for Arbitrary Masses	182
	Appendix 4: Hydrodynamic Equations	186
	Appendix 5: Alternative Derivation of the Vanishing Bulk Viscosity	186
	Appendix 6: n -Body Resonances	187
	References	189
6	Universal Relations for Fermions with Large Scattering Length.	193
6.1	Introduction	193
6.2	The Tan Relations	194
6.2.1	Tails of Distributions	194
6.2.2	Changes in the Scattering Length.	196
6.2.3	Additional Tan Relations.	197
6.3	What is the Contact?.	198
6.3.1	Intuitive Interpretation.	198
6.3.2	Few-Body Systems	199
6.3.3	Balanced Homogeneous Gas	200
6.3.4	Strongly-Imbalanced Homogeneous Gas	203
6.4	Other Universal Relations	205
6.4.1	RF Spectroscopy	205
6.4.2	Photoassociation.	206
6.4.3	Structure Factors	207
6.4.4	Viscosity Spectral Functions	208
6.4.5	Rapid Change in the Scattering Length.	209
6.5	Making Contact with Experiment.	210
6.5.1	Photoassociation.	210
6.5.2	Static Structure Factor.	211
6.5.3	Comparing Measurements of the Contact	212
6.5.4	Tests of the Thermodynamic Tan Relations.	213
6.5.5	Contact Density Near Unitarity	214

6.6	Derivations of Universal Relations	215
6.6.1	Preliminaries	215
6.6.2	Tan's Derivations	217
6.6.3	Other Derivations	218
6.6.4	Quantum Field Theory Derivations	221
	References	230
7	Unitary Fermi Gas, ϵ Expansion, and Nonrelativistic Conformal Field Theories	233
7.1	Introduction	233
7.2	ϵ Expansion for the Unitary Fermi Gas	236
7.2.1	Why Four and Two Spatial Dimensions are Special?	236
7.2.2	Feynman Rules and Power Counting of ϵ	239
7.2.3	Zero Temperature Thermodynamics	242
7.2.4	Quasiparticle Spectrum	247
7.2.5	Critical Temperature	248
7.2.6	Phase Diagram of an Imbalanced Fermi Gas	251
7.3	Aspects as a Nonrelativistic Conformal Field Theory	251
7.3.1	Schrödinger Algebra	251
7.3.2	Operator-State Correspondence	252
7.3.3	Scaling Dimensions of Composite Operators	257
7.3.4	Application of the ϵ Expansion	260
7.4	General Coordinate and Conformal Invariance	265
7.4.1	Vanishing Bulk Viscosities	267
7.4.2	Superfluid Effective Field Theory	270
7.5	Other Scale and Conformal Invariant Systems	271
	Appendix: Scaling Dimensions of Three-Body Operators	272
	References	274
8	Dilute Fermi and Bose Gases	277
8.1	Introduction	277
8.2	The Dilute Spinless Fermi Gas	280
8.2.1	Dilute Classical Gas, $k_B T \ll \mu $, $\mu < 0$	282
8.2.2	Fermi Liquid, $k_B T \ll \mu$, $\mu > 0$	283
8.2.3	High- T Limit, $k_B T \gg \mu $	284
8.3	The Dilute Bose Gas	286
8.3.1	$d < 2$	288
8.3.2	$d = 3$	291
8.4	The Dilute Spinful Fermi Gas: The Feshbach Resonance	291
8.4.1	The Fermi-Bose Model	295
8.4.2	Large N Expansion	299
	References	304

9	The Unitary Fermi Gas: From Monte Carlo to Density Functionals	305
9.1	Introduction	305
9.2	The Quantum Monte Carlo Approach	308
9.2.1	From the Physical Problem to the Lattice Formulation	308
9.2.2	Effective Hamiltonian	309
9.2.3	The Hubbard–Stratonovich Transformation	310
9.2.4	Stabilization of the Algorithm for Small Temperatures	316
9.2.5	Finite Size Scaling	318
9.2.6	Results: The Energy and the Entropy	319
9.2.7	Response to External Probes and the Spectral Function	323
9.2.8	The Pairing Gap, Pseudogap, and Critical Temperature.	325
9.2.9	Describing Trapped Systems with Quantum Monte Carlo Results	327
9.3	Density Functional Theory for the Unitary Fermi Gas.	330
9.3.1	The Energy Density Functional	333
9.3.2	Determining the SLDA and ASLDA Energy Density Functionals	339
9.3.3	Using the SLDA and ASLDA	346
9.4	Time-Dependent Superfluid Local Density Approximation	350
9.4.1	Time-Dependent Equations for the Quasiparticle Wave Functions	350
9.4.2	Galilean Invariance.	352
9.4.3	The Excitation of the Pairing Higgs Mode.	355
9.4.4	Generation and Dynamics of Vortices.	358
9.5	Concluding Remarks.	362
	Appendix	363
	References	369
10	Scaling Flows and Dissipation in the Dilute Fermi Gas at Unitarity	375
10.1	Introduction	375
10.2	Scaling Flows	376
10.3	Elliptic Flow	379
10.3.1	Energy Dissipation	380
10.3.2	Moments of the Navier–Stokes Equation.	383
10.3.3	Scaling Solution of the Navier–Stokes Equation.	384
10.3.4	Breakdown of Hydrodynamics	388
10.3.5	Relaxation Time Approach	390

10.4	Expansion From a Rotating Trap	392
10.4.1	Ideal Fluid Dynamics	392
10.4.2	Dissipation.	395
10.5	Collective Oscillations	398
10.6	Summary and Outlook	402
	References	405
11	Thermodynamics of Fermi Gases	407
11.1	Introduction	407
11.2	Thermodynamics of Strongly Interacting Fermi Gases	409
11.2.1	Thermodynamics in the Grand Canonical Ensemble and the Local Density Approximation	409
11.2.2	Measuring the Local Pressure of a Trapped Fermi Gas	411
11.3	The Zero Temperature Phase Diagram	412
11.3.1	The BEC–BCS Crossover	412
11.3.2	Spin Imbalanced Gases and the Clogston–Chandrasekhar Limit.	415
11.4	Beyond the Clogston–Chandrasekhar Limit	420
11.4.1	The Impurity Problem and the Polaron/Molecule Transition	421
11.4.2	The Polarized Superfluid	423
11.5	The Normal Phase	424
11.5.1	Landau’s Fermi Liquid Theory.	424
11.5.2	The Gas of Polarons	425
11.5.3	The Fermi Liquid in the Unpolarized Limit.	428
11.5.4	The Normal/Superfluid Transition Line.	430
11.5.5	The Tricritical Point	432
11.6	Fermi Liquid Versus Non-Fermi Liquid	433
11.6.1	Molecular Limit	433
11.6.2	Pseudogap	434
11.7	High Temperature Expansion.	435
11.8	Conclusion.	436
	Appendix	437
	References	443
12	Normal Phase of Polarised Strongly Interacting Fermi Gases.	447
12.1	Introduction	447
12.2	The $N + 1$ or Polaron Problem	449
12.2.1	Homogeneous Case.	449
12.2.2	Non-Homogeneous/Trapped Case	452

12.3	Unbalanced Fermi Gas or Many-Polaron Problem	453
12.3.1	Homogeneous Case.	453
12.3.2	Non-Homogeneous/Trapped Case.	456
12.3.3	Different Mass Case: Fermi Mixture.	458
12.4	RF Spectroscopy	463
12.5	Collisional Properties of the Normal Phase	465
12.5.1	$T = 0$	467
12.5.2	$T \neq 0$	467
12.5.3	Experimental Consequences for Collective Modes	468
12.6	Collective Oscillations: The Quadrupole Mode	470
	References	475
13	Thermodynamics of Trapped Imbalanced Fermi	
	Gases at Unitarity	477
13.1	Introduction	477
13.2	Ultracold Quantum Fields	479
13.2.1	Bardeen-Cooper-Schrieffer Theory	479
13.2.2	Fluctuations	481
13.3	The Thermodynamic Potential	484
13.3.1	Normal State	485
13.3.2	Superfluid State	488
13.4	Applications.	491
13.4.1	Interface and Surface Tension	492
13.4.2	Deformation.	495
13.5	Conclusions	501
	References	502
14	BCS–BEC Crossover and Unconventional Superfluid	
	Order in One Dimension	503
14.1	Introduction	504
14.2	BCS–BEC Crossover of a Balanced One-Dimensional Fermi Gas	505
14.3	Spin-Imbalanced Fermi Gas in One Dimension	509
14.3.1	Bethe-Ansatz Solution for the Homogeneous Gas.	510
14.3.2	Phase Separation in a Trap	512
14.4	FFLO Correlations in the Partially Polarized Phase	513
14.4.1	Predictions From Bosonization.	513
14.4.2	Exact Numerical Results	516
14.5	Spin- and Mass-Imbalanced Fermi Mixtures	520
14.6	A Two-Channel Model: The Bose-Fermi Resonance Model	523
14.6.1	Phase Diagram of the Bose-Fermi Resonance Model at Finite Imbalance.	524

14.7 Beyond 1D, Other Variations 526

14.8 Proposals for the Experimental Observation
of FFLO Correlations 528

14.9 Summary 530

References 530