

A Mathematical Theory of the Mechanical Behavior of Continuous Media

WALTER NOLL

Communicated by C. TRUESDELL

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A New Mathematical Theory of Simple Materials

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Dedicated to Clifford Truesdell, friend and mentor of the author

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