

## CONTENTS

### AN INTRODUCTION TO STOCHASTIC PROCESSES FOR PHYSICISTS

N.G. van KAMPEN

|                                              |    |
|----------------------------------------------|----|
| 1. Preliminaries                             | 2  |
| 2. Stochastic functions                      | 4  |
| 3. Some important Markov processes           | 7  |
| 4. The master equation                       | 10 |
| 5. The $\Omega$ -expansion                   | 12 |
| 6. Limitation of the $\Omega$ -expansion     | 15 |
| 7. Calculation of the escape rate            | 17 |
| 8. A first-passage problem in two dimensions | 20 |
| References                                   | 23 |

### STOCHASTIC DIFFERENTIAL EQUATIONS WITH NON-MARKOV PROCESSES

E. SANTOS

|                                           |    |
|-------------------------------------------|----|
| 1. Introduction                           | 25 |
| 2. Markov and non-Markov processes        | 25 |
| 3. Gaussian and stationary processes      | 29 |
| 4. Stochastic differential equations      | 32 |
| 5. Harmonic analysis                      | 34 |
| 6. An example of the Markov limit         | 36 |
| 7. The approximate Fokker-Planck equation | 41 |
| 8. Expansions in the correlation time     | 44 |
| 9. Integration in functional spaces       | 47 |
| References                                | 51 |

### ASPECTS OF THE THEORY OF BROWNIAN MOTION

R.M. MAZO

|                                  |    |
|----------------------------------|----|
| 1. Introduction                  | 54 |
| 2. Dynamical theory - stochastic | 56 |
| 3. Diffusion                     | 62 |
| 4. Projection-operator methods   | 64 |
| 5. Dynamical theory - molecular  | 67 |
| 6. Concentration dependence      | 72 |
| 7. Nonuniform systems            | 75 |
| 8. Concluding remarks            | 78 |
| References                       | 81 |

### PATH-INTEGRAL METHODS IN NONEQUILIBRIUM THERMODYNAMICS AND STATISTICS

R. GRAHAM

|                                                                                |    |
|--------------------------------------------------------------------------------|----|
| 1. Introduction                                                                | 83 |
| 2. Basic postulates and equations of statistical nonequilibrium thermodynamics | 85 |

## VI

|                                                                                                      |     |
|------------------------------------------------------------------------------------------------------|-----|
| 3. Linear processes                                                                                  | 90  |
| 4. Variational principles and probability densities for linear processes                             | 99  |
| 5. Path-integral solution of the Fokker-Planck equation                                              | 106 |
| 6. The Lagrangian and dissipative potentials                                                         | 113 |
| 7. The Onsager-Machlup function for nonlinear processes                                              | 117 |
| 8. Relation of the Onsager-Machlup function and the path-integral solution                           | 126 |
| 9. Application to the Bénard problem                                                                 | 128 |
| Appendix: Derivation of the probability-density functional from the maximum-entropy principle        | 135 |
| References                                                                                           | 137 |
| SYNERGETICS - A FIELD BEYOND IRREVERSIBLE THERMODYNAMICS <span style="float: right;">H. HAKEN</span> |     |
| 1. Introduction                                                                                      | 140 |
| 2. Thermodynamics, irreversible thermodynamics, synergetics                                          | 140 |
| 3. Examples                                                                                          | 143 |
| 4. Self-organization in continuously extended media                                                  | 150 |
| 5. Generalized Ginzburg-Landau equations for nonequilibrium phase transitions                        | 151 |
| 6. An application of generalized Ginzburg-Landau equations to morphogenesis in biology               | 158 |
| 7. Nonequilibrium phase transitions and bifurcation of limit cycles                                  | 165 |
| 8. Concluding remarks                                                                                | 166 |
| References                                                                                           | 167 |
| COMPUTER RESULTS ON TRANSPORT PROPERTIES <span style="float: right;">B.J. ALDER</span>               |     |
| 1. General remarks                                                                                   | 169 |
| 2. The velocity autocorrelation function                                                             | 171 |
| 3. Fluctuation-driven resonance                                                                      | 173 |
| 4. The neutron scattering function                                                                   | 177 |
| 5. Generalized hydrodynamics                                                                         | 182 |
| 6. The Lorentz gas                                                                                   | 186 |
| 7. Waiting-time distribution                                                                         | 188 |
| 8. Results                                                                                           | 191 |
| References                                                                                           | 209 |
| KINETICS OF PHASE TRANSITIONS <span style="float: right;">O. PENROSE</span>                          |     |
| 1. Introduction                                                                                      | 211 |
| 2. Metastable states in mean-field theory                                                            | 214 |
| 3. Ensembles for metastable states                                                                   | 219 |
| 4. The theory of nucleation                                                                          | 222 |

## VII

|                                |     |
|--------------------------------|-----|
| 5. Growth in the lattice model | 228 |
| 6. Coarsening                  | 231 |
| References                     | 234 |

|                                                 |           |
|-------------------------------------------------|-----------|
| STOCHASTIC BEHAVIOR OF SIMPLE DYNAMICAL SYSTEMS | Y. POMEAU |
|-------------------------------------------------|-----------|

|                                                                                      |     |
|--------------------------------------------------------------------------------------|-----|
| 1. Introduction                                                                      | 236 |
| 2. Measure space - dynamical systems                                                 | 238 |
| 3. Ergodic systems. The Birkhoff and Oseledec theorems                               | 244 |
| 4. Weak and strong mixing: the Kakutani counter example                              | 256 |
| 5. Bernoulli shifts: The Meshalkin isomorphism                                       | 264 |
| 6. Strange attractors, iteration of mappings of the plane and of the line, and so on | 269 |
| References                                                                           | 275 |

### SEMINARS AND SHORT COMMUNICATIONS

|                                                                                                                |     |
|----------------------------------------------------------------------------------------------------------------|-----|
| FOKKER-PLANCK DESCRIPTION OF A NAVIER-STOKES FLUID WITH INSTABILITY (M. DROZ, C.P. ENZ and A. MALASPINAS)      | 280 |
| OPERATOR FORMALISM FOR CONTINUOUS MARKOV PROCESSES (L. GARRIDO and M. SAN MIGUEL)                              | 287 |
| APPLICATIONS OF PATH INTEGRAL (E.P. GROSS)                                                                     | 300 |
| FUNCTIONAL INTEGRAL METHODS FOR RANDOM FIELDS (F. LANGOUCHE, D. ROEKAERTS and E. TIRAPEGUI)                    | 316 |
| KINETIC EQUATIONS FROM HAMILTONIAN DYNAMICS: THE MARKOVIAN LIMIT (H. SPOHN)                                    | 330 |
| BOLTZMANN BEHAVIOUR OF A SPATIALLY INHOMOGENEOUS GAS (J. BIEL and J. MARRO)                                    | 336 |
| AN EXAMPLE OF PROCESSES IN PHYSICS THAT ARE NONLOCAL IN TIME (A. GERSCHEL)                                     | 337 |
| STOCHASTIC THEORY FOR HYDRODYNAMICAL SYSTEMS (CH. VAN DEN BROECK and L. BRENIG)                                | 338 |
| FLUCTUATION SPECTRA NEAR THE THRESHOLD OF A CURRENT INSTABILITY (M. BUTTIKER and H. THOMAS)                    | 339 |
| STOCHASTIC ELECTRODYNAMICS (P. CLAVERIE)                                                                       | 340 |
| BROWNIAN MOTION IN PERSISTENT FLUCTUATIONS (R. GRAPPIN)                                                        | 341 |
| COVARIANT WIGNER-FUNCTION APPROACH TO RELATIVISTIC QUANTUM STATISTICS (R. HAKIM)                               | 342 |
| STOCHASTIC MODEL OF CURRENT-COUPLED CHANNELS IN NERVE MEMBRANES (I. v.d. HEYDT, N. v.d. HEYDT and G. OBERMAIR) | 343 |
| DERIVATION OF STOCHASTIC TRANSPORT EQUATIONS FOR SYSTEMS OF INTERACTING BROWNIAN PARTICLES (W. HESS)           | 344 |
| THERMODYNAMIC INSTABILITY, OSCILLATIONS, AND MODIFICATION OF NERVE SYNAPSES (K. KAUFMANN)                      | 345 |
| DISCUSSION OF A RANDOM-WALK MODEL WITH CORRELATED JUMPS (J.W. HAUS and K.W. KEHR)                              | 346 |

# VIII

|                                                                                                     |     |
|-----------------------------------------------------------------------------------------------------|-----|
| DISCUSSION OF A QUANTUM THEORY OF DIFFUSION INCLUDING MEMORY EFFECTS (KAZUO KITAHARA and J.W. HAUS) | 347 |
| THE DYNAMO EFFECT IN MAGNETHYDRODYNAMIC TURBULENCE (J. LEORAT)                                      | 348 |
| DYNAMICAL CORRELATIONS IN NONLINEAR CHEMICAL REACTIONS (R. SCHRANNER and S. GROSSMANN)              | 349 |
| A COOPERATIVE EFFECT WITH SIGNIFICANCE IN BIOLOGICAL EVOLUTION (J. WAGENSBERG)                      | 350 |