

Contents

Introduction	1
1 Thom-Sebastiani Theorem for constructible sheaves	17
Introduction	17
1.1 Milnor fibration	23
1.1.1 Cohomological version of a Milnor fibration	24
1.1.2 Examples	32
1.2 Thom-Sebastiani Theorem	38
1.2.1 Preliminaries and Thom-Sebastiani for additive functions	38
1.2.2 Thom-Sebastiani Theorem for sheaves	50
1.3 The Thom-Sebastiani Isomorphism in the derived category	62
1.4 Appendix: Künneth formula	74
2 Constructible sheaves in geometric categories	81
Introduction	81
2.0.1 The basic results	81
2.0.2 Definable spaces	89
2.1 Geometric categories	93
2.2 Constructible sheaves	98
2.3 Constructible functions	122
3 Localization results for equivariant constructible sheaves	141
Introduction	141
3.1 Equivariant sheaves	149
3.1.1 Equivariant sheaves and monodromic complexes	149
3.1.2 Equivariant derived categories	155
3.1.3 Examples and stalk formulae	159
3.2 Localization results for additive functions	170
3.3 Localization results for Grothendieck groups and trace formulae	176
3.3.1 Grothendieck groups	176
3.3.2 Trace formulae	192
3.4 Equivariant cohomology	200

4	Stratification theory and constructible sheaves	207
	Introduction	207
4.1	Stratification theory	217
4.1.1	A cohomological version of the first isotopy lemma	217
4.1.2	Comparison of different regularity conditions	223
4.1.3	Micro-local characterization of constructible sheaves	229
4.2	Constructible sheaves on stratified spaces	232
4.2.1	Cohomologically cone-like stratifications	232
4.2.2	Stability results for constructible sheaves	241
4.3	Base change properties	252
4.3.1	Some constructions for stratifications	253
4.3.2	Base change isomorphisms	259
5	Morse theory for constructible sheaves	269
	Introduction	269
5.0.1	Real stratified Morse theory	270
5.0.2	Complex stratified Morse theory	281
5.0.3	Introduction to characteristic cycles	283
5.1	Stratified Morse theory, part I	296
5.1.1	Local Morse data	296
5.1.2	Normal Morse data	307
5.1.3	Morse theory for a stratified space with corners	316
5.2	Characteristic cycles and index formulae	320
5.2.1	Index formulae and Euler obstruction	321
5.2.2	A specialization argument	328
5.3	Stratified Morse theory, part II	334
5.3.1	Normal Morse data are independent of choices	334
5.3.2	Splitting of the local Morse data	345
5.3.3	Normal Morse data and micro-localization	355
5.4	Vanishing cycles	361
6	Vanishing theorems for constructible sheaves	375
	Introduction: Results and examples	375
6.0.1	(Co)stalk properties	375
6.0.2	Intersection (co)homology and perverse sheaves	381
6.0.3	Vanishing results in the complex context	385
6.0.4	Nearby and vanishing cycles	402
6.0.5	Artin-Grothendieck type theorems	408
6.0.6	Applications to constructible functions	412
6.1	Proof of the results	414
	Bibliography	433
	Index	447