Contents

1	NS-	_	ations and Modelling: A French Touch		
	1.1		irst Contribution to the RAM Approach		
			id Dynamics	3	
	1.2		ollaboration with Jean-Pierre Guiraud in the Aerodynamics		
	_	•	tment of ONERA	8	
	1.3		v Remarks Concerning My Preceding Books on Modelling		
			wtonian Fluid Flows	10	
	1.4	Concl	usion	14	
Pa	ırt I	Navie	r-Stokes-Fourier Equations: General Main Models		
2	New	tonian	Fluid Dynamics as a Mathematical – Physical Science	19	
	2.1	From	Newton to Euler	21	
		2.1.1	Eulerian Elastic Fluid	21	
		2.1.2	From Adiabaticity to Isochoricity	23	
	2.2	Navie	r Viscous Incompressible, Constant Density Equations	25	
	2.3 Navier–Stokes–Fourier Equations for Viscous Con		r-Stokes-Fourier Equations for Viscous Compressible		
		and Heat-Conducting Fluid Flow			
		2.3.1	The Cauchy Stress Principle	28	
		2.3.2	Navier–Stokes Constitutive Equations:		
			The Cauchy–Poisson Law	29	
		2.3.3	Thermodynamics and Energy Equation via Fourier		
			Constitutive Equation	32	
		2.3.4	Navier-Stokes-Fourier (NS-F) Equations	34	
	2.4	Initial	and Boundary Conditions	35	
		2.4.1	The Problem of Initial Conditions	36	
		2.4.2	Unsteady Adjustment Problems	38	
		2.4.3	Boundary Conditions for the Velocity Vector		
			u and Temperature T	4(
		2.4.4	Other Types of Boundary Conditions	41	
			•		



xiii

xiv Contents

3	Froi		F Equations to General Main Model Equations	51	
	3.1	Non-d	limensional Form of the NS-F Typical Problem	52	
		3.1.1	Non-dimensional NS-F Equations and Reduced		
			Parameters	52	
		3.1.2	Conditions for the Unsteady NS–F Equations	57 59	
	3.2	Gener	General Main Model Equations		
		3.2.1	Some General Main Models	59	
		3.2.2	A Sketch of the Various General Main Models	61	
	3.3	Non-uniform Validity of Main Model Equations and the Local			
		Limiting Processes			
		3.3.1	Large Reynolds Case: A First Naive Approach	63	
		3.3.2	Low Mach Number Case: The Navier System of		
			Equations and Companion Acoustics Equations	65	
		3.3.3	Oberbeck-Boussinesq Model Equations for the		
			Rayleigh-Bénard Shallow Thermal Convection	69	
		3.3.4	Stokes-Oseen Model Equations in the Case of a Low		
			Reynolds Number	71	
		3.3.5	The Case of Non-linear Acoustics	76	
		3.3.6	A Sketch of Consistent Models Derived from NS-F		
			Equations Relative to the Considered Reference		
			Parameters	80	
4	ΑT	ypical l	RAM Approach: Boussinesq Model Equations	81	
	4.1		luctory Remarks Concerning Atmospheric Flow	81	
	4.2			84	
		4.2.1	Euler Dimensionless Equations for the Thermodynamic		
			Perturbations	85	
		4.2.2	Rational Derivation of Boussinesq Equations	86	
		4.2.3	Two Particular Cases	89	
	4.3	The S	steady 2D Case	90	
	4.4		Problem of Initial Conditions	92	
	4.5	A Ske	etch of a RAM Approach for a Boussinesq Model	94	
Pa	rt II	A Sk	etch of a Mathematical Theory of the RAM Approach		
5	The	Struct	ture of Unsteady NS–F Equations at Large		
	Rey	Reynolds Numbers			
	5.1	Introd	luction	97	
	5.2		Emergence of the Four Regions as a Consequence		
		of the Singular Nature of BL Equations Near $t = 0 \dots 9$			
	5.3		ulation of the Unsteady NS-F Problem	100	
	5.4		ation of the Corresponding Four Model Problems	103	
		5.4.1	Euler-Prandtl Regular Coupling	103	

Contents

		5.4.2	Acoustic and Rayleigh Problems Near	
			the Initial Time $t = 0 \dots \dots \dots$	107
	5.5	-	ment Processes Towards the Prandtl BL Evolution	
		Proble	m	110
		5.5.1	Adjustment Process Via the Acoustics/Gas Dynamics	
			Equations	111
		5.5.2	Adjustment Process Via the Rayleigh Equations	112
	5.6	Some	Conclusions	114
6	The	Mathe	matics of the RAM Approach	117
•	6.1	Our B	asic Postulate for the Realization of the RAM Approach.	117
	6.2		lathematical Nature of NS-F Equations:	
	٠		id-Dynamical Point of View	120
	6.3		lation of Dimensionless Equations for Applications	
			RAM Approach	124
		6.3.1	Turbomachinery Fluid Flows	124
		6.3.2	The G–Z "Rolled-up Vortex Sheet" Theory: Vortex	
			Sheets and Concentrated Vorticity	127
		6.3.3	The G-Z Asymptotic Approach to Non-linear	
			Hydrodynamic Stability	130
		6.3.4	A Local Atmospheric Thermal Problem: A Triple-Deck	
			Viewpoint	135
		6.3.5	Miscellanea	138
	6.4	Some	Key Steps for the Application of the Basic Postulate	138
		6.4.1	Similarity Rules: Small Mach and Large Reynolds	
			Numbers Flow	140
		6.4.2	The Matching Principle	141
		6.4.3	The Least-Degeneracy Principle and the Significant	
			Degeneracies	144
		6.4.4	The Method of Multiple Scales (MMS)	145
		6.4.5	The Homogenization Analysis	149
		6.4.6	Asymptotics of the Triple-Deck Theory	154
Pa	art III	App	lications of the RAM Approach to Aerodynamics, Thermal	
_			vection, and Atmospheric Motions	
_	m.	D . 3.7		161
7			Approach in Aerodynamics	101
	7.1		ation of a Through-Flow Model Problem for Fluid Flow	161
			Axial Compressor	
		7.1.1	The Veuillot Approach	161
		7.1.2	The G–Z Approach	165
		7.1.3	Transmission Conditions, Local Solution at the	1.00
			Leading/Trailing Edges, and Matching	168
		7.1.4	Some Complements	169
	7.2		Flow Within a Cavity Which is Changing Its Shape and	
		Volur	me with Time: Low Mach Number Limiting Case	171

xvi Contents

		7.2.1	Formulation of the Inviscid Problem	172
		7.2.2	The Persistence of Acoustic Oscillations	173
		7.2.3	Derivation of an Average Continuity Equation	174
		7.2.4	Solution for the Fluctuations \mathbf{u}_0^* and ρ_1^*	178
		7.2.5	The Second-Order Approximation	179
		7.2.6	The Average System of Equations for the Slow	
			Variation	182
		7.2.7	The Long Time Evolution of the Fast Oscillations	184
		7.2.8	Some Concluding Comments	187
8	The	RAM A	Approach in the Bénard Convection Problem	193
-	8.1		troduction	193
	8.2		Unexpected Results for the Bénard Problem of an	
	٠.٣		sible Liquid Layer Heated from Below	194
		8.2.1	The Questionable Davis (1987) Upper, Free Surface,	
		0.2.1	Temperature Condition, and the Problem	
			of Two Biot Numbers	196
		8.2.2	The Mystery of the Disappearance of Influence	
		0.2.2	of the Free Surface in the RB Leading-Order Shallow	
			Thermal Convection Model	199
		8.2.3	Influence of the Viscous Dissipation	201
	8.3		Marangoni Effect	203
	0.2	8.3.1	The Long-Wave Approach	205
		8.3.2	Towards a Lubrication Equation	208
	8.4		Deep to Shallow Thermal Convection Model Problems	210
9	The	RAM	Approach in Atmospheric Motions	213
	9.1		Models for the Lee-Waves Problem	213
	· · ·	9.1.1	First Integrals	214
		9.1.2	An Equation for the Vertical Deviation $\delta(x, z)$	215
		9.1.3	Dimensionless Formulation	218
		9.1.4	Four Distinguished Limiting Cases	220
	9.2		ow Kibel Number Asymptotic Model	223
	,	9.2.1	The Dissipative (Viscous and Non-adiabatic) NS-F	
			atmospheric Equations	226
		9.2.2	Hydrostatic Limiting Processes	229
		9.2.3	The Dissipative Hydrostatic Equations in p-System	233
		9.2.4	The Leading-Order QG Model Problem	237
		9.2.5	The Second-Order Ageostrophic G–Z Model	247
		9.2.6	Kibel Primitive Equations and Lee-Waves Problems	
			as Inner and Outer Asymptotic Models	248
E	pilog	ue		253
R	efere	nces		261
Īr	ndex .			269