

Contents

1	Conservation laws	1
1.1	Fundamental principles	1
1.2	Scalar conservation laws	3
	Transport phenomena	3
	Weak solutions	7
	The Riemann problem	9
1.3	Systems of conservation laws	15
	Systems in several space dimensions	15
	Symmetric systems	25
	Entropy solutions	28
2	Numerical approximations for conservation laws	31
2.1	Basic concepts	31
	Monotone schemes and TVD formulation	34
	Incremental form and numerical viscosity	37
	Examples of some classical schemes	39
2.2	Entropy solutions	45
	Consistency	46
	E-schemes	47
	Discrete entropy inequalities	48
	Discrete cell entropy inequality	53
3	Dissipation filters	61

3.1	Linear filters	63
	Gaussian smoothing	63
	Linear diffusion equations	64
3.2	Nonlinear diffusion filters	66
	The Perona-Malik model	66
	TV-preserving models	73
	Anisotropic filter models	75
4	Discrete filters for scalar conservation laws	79
4.1	The basic filter	79
	The convective step	81
	The dissipation step	81
	The resulting algorithm	89
4.2	Data dependent diffusion steering	89
	Coherence measure	90
	Shock strength measures	92
4.3	Entropy based filter	93
	Entropy-steered diffusion	93
	Diffusion tensor for smooth solutions	94
	Diffusion tensor near discontinuities	96
4.4	Positive filter	97
	The basic scheme	97
	Positivity conditions	98
	The positive filter	101
5	Discrete filters for the Euler equations	103
5.1	The Euler equations	103
5.2	Characteristic filters	105
	Construction of the characteristic filter	105
	The structure tensor	107

The diffusion matrix	108
6 Numerical examples	111
6.1 Scalar test case	111
Basic scheme with coherence measure	112
Weighted coherence measure	116
The entropy controlled blending scheme	118
Positive dissipation schemes	120
Order of convergence	122
6.2 Euler test case	123
7 Conclusions and perspectives	127
Bibliography	129
Zusammenfassung	136