

Contents

Preface	v
Introduction	1
Glossary	3
1 Heegaard splittings	16
1.1 Introduction	16
1.2 Existence of Heegaard splittings	17
1.3 Stable equivalence of Heegaard splittings	18
1.4 The mapping class group	21
1.5 Manifolds of Heegaard genus ≤ 1	23
1.6 Seifert manifolds	26
1.7 Heegaard diagrams	28
1.8 Exercises	31
2 Dehn surgery	32
2.1 Knots and links in 3-manifolds	32
2.2 Surgery on links in S^3	33
2.3 Surgery description of lens spaces and Seifert manifolds	35
2.4 Surgery and 4-manifolds	39
2.5 Exercises	42
3 Kirby calculus	43
3.1 The linking number	43
3.2 Kirby moves	45
3.3 The linking matrix	54
3.4 Reversing orientation	55
3.5 Exercises	56
4 Even surgeries	58
4.1 Exercises	62
5 Review of 4-manifolds	63
5.1 Definition of the intersection form	63
5.2 The unimodular integral forms	67
5.3 Four-manifolds and intersection forms	68
5.4 Exercises	71

6	Four-manifolds with boundary	72
6.1	The intersection form	72
6.2	Homology spheres via surgery on knots	77
6.3	Seifert homology spheres	77
6.4	The Rohlin invariant	79
6.5	Exercises	80
7	Invariants of knots and links	81
7.1	Seifert surfaces	81
7.2	Seifert matrices	83
7.3	The Alexander polynomial	85
7.4	Other invariants from Seifert surfaces	89
7.5	Knots in homology spheres	91
7.6	Boundary links and the Alexander polynomial	93
7.7	Exercises	96
8	Fibered knots	98
8.1	The definition of a fibered knot	98
8.2	The monodromy	100
8.3	More about torus knots	102
8.4	Joins	103
8.5	The monodromy of torus knots	105
8.6	Open book decompositions	106
8.7	Exercises	108
9	The Arf-invariant	109
9.1	The Arf-invariant of a quadratic form	109
9.2	The Arf-invariant of a knot	112
9.3	Exercises	115
10	Rohlin's theorem	116
10.1	Characteristic surfaces	116
10.2	The definition of $\tilde{q}$	117
10.3	Representing homology classes by surfaces	122
11	The Rohlin invariant	123
11.1	Definition of the Rohlin invariant	123
11.2	The Rohlin invariant of Seifert spheres	123
11.3	A surgery formula for the Rohlin invariant	127
11.4	The homology cobordism group	129
11.5	Exercises	133

12 The Casson invariant	135
12.1 Exercises	141
13 The group $SU(2)$	142
13.1 Exercises	147
14 Representation spaces	148
14.1 The topology of representation spaces	148
14.2 Irreducible representations	149
14.3 Representations of free groups	150
14.4 Representations of surface groups	150
14.5 Representations for Seifert homology spheres	153
14.6 Exercises	158
15 The local properties of representation spaces	159
15.1 Exercises	162
16 Casson's invariant for Heegaard splittings	163
16.1 The intersection product	163
16.2 The orientations	166
16.3 Independence of Heegaard splitting	168
16.4 Exercises	171
17 Casson's invariant for knots	172
17.1 Preferred Heegaard splittings	172
17.2 The Casson invariant for knots	173
17.3 The difference cycle	177
17.4 The Casson invariant for boundary links	178
17.5 The Casson invariant of a trefoil	179
18 An application of the Casson invariant	181
18.1 Triangulating 4-manifolds	181
18.2 Higher-dimensional manifolds	182
18.3 Exercises	183
19 The Casson invariant of Seifert manifolds	184
19.1 The space $\mathcal{R}(p, q, r)$	184
19.2 Calculation of the Casson invariant	187
19.3 Exercises	190
Conclusion	191
Bibliography	195
Index	205