

Contents

1	Origins and Concepts	1
1.1	Introduction	1
1.2	Magnetism	2
1.3	Gravitation	3
1.4	Faraday, Thomson, and Maxwell	3
1.5	Gravitation a Vector Field	4
1.6	Charges and Electric Fields	5
1.7	Priestly's Speculation	6
1.8	Voltaic Cell	7
1.9	Currents and Magnetic Fields	8
1.9.1	Oersted	8
1.9.2	Ampère	10
1.9.3	Electrical current	11
1.10	Induced Electric Field	12
1.11	The Mathematical Theory	13
1.11.1	The field equations	13
1.11.2	Maxwell	15
1.12	Experimental Evidence	20
1.12.1	Waves in the laboratory	20
1.12.2	Wave energy and momentum	23
1.13	Michelson and Morley Experiment	24
1.14	Relativity	27
1.15	Summary	29
	Questions	30
2	Mathematical Background	33
2.1	Introduction	33
2.2	Vectors	34
2.2.1	The Vector Space	34
2.2.2	Representation	36
2.2.3	Scalar Product	39
2.2.4	Vector Product	40

2.3	Multivariate Functions	41
2.3.1	Differentials	42
2.3.2	Cylindrical Coordinates	43
2.3.3	Spherical Coordinates	44
2.4	Analytic Functions	45
2.4.1	Taylor Series	45
2.4.2	Analyticity	46
2.5	Vector Calculus	46
2.5.1	Field Quantities	46
2.5.2	The Gradient	47
2.5.3	The Divergence	48
2.5.4	The Curl	54
2.5.5	The Laplacian Operator	61
2.6	Differential Equations	62
2.6.1	Helmholtz' Theorem.....	64
2.6.2	The Del Operator	65
2.6.3	Dirac Delta Function.....	66
2.7	Summary	71
	Exercises	72
3	Electrostatics	79
3.1	Introduction	79
3.2	Coulomb's Law	79
3.2.1	Coulomb's Experiment	79
3.2.2	Units	81
3.3	Superposition.....	81
3.4	Distributions of Charges	82
3.4.1	Distribution of Point Charges	83
3.4.2	Volume Charge Density	84
3.4.3	Surface Charge Density.....	85
3.5	The Field Concept	87
3.6	Divergence and Curl of $\mathbf{E}$	89
3.7	Integral Electrostatic Field Equations.....	91
3.7.1	Gauss' Theorem.....	92
3.7.2	Stokes' Theorem	93
3.8	Summary	93
	Exercises	94
4	The Scalar Potential	99
4.1	Introduction	99
4.2	Potential Energy	99
4.3	Potential Surfaces	101
4.4	Poisson's Equation.....	101
4.5	Multipole Expansion.....	108
4.6	Energy Storage.....	110
4.6.1	Electrostatic Energy Density	110
4.6.2	Energy of a Set of Conductors	111

4.7	Summary	116
	Exercises	117
5	Magnetostatics	123
5.1	Introduction	123
5.2	Current	124
5.2.1	Current Density	124
5.2.2	Charge Conservation	125
5.3	Oersted's Experiment	126
5.4	Ampère's Experiment	129
5.4.1	Direction of the Force	130
5.4.2	The Constant	131
5.5	Consequences of Ampère's Experiment	132
5.5.1	Force on a Charge	132
5.5.2	Field from a Straight Wire	133
5.5.3	Biot-Savart Law	134
5.6	Superposition	136
5.7	Multipole Expansion	138
5.8	Divergence and Curl of $\mathbf{B}$	141
5.9	Gauge Transformation	142
5.10	The Static Field Equations	143
5.11	Summary	145
	Exercises	145
6	Applications of Magnetostatics	149
6.1	Introduction	149
6.2	Biot-Savart Law	149
6.3	Vector Potential	151
6.4	Summary	159
	Exercises	159
7	Particle Motion	165
7.1	Introduction	165
7.2	Analytical Mechanics	165
7.2.1	Euler-Lagrange Formulation	165
7.2.2	Hamiltonian Formulation	166
7.3	Electrodynamics	167
7.3.1	The Lagrangian	167
7.3.2	The Hamiltonian	169
7.4	Particle Motion	169
7.4.1	Magnetic Fields	169
7.4.2	Electric and Magnetic Fields	176
7.5	Plasmas	180
7.6	Summary	182
	Exercises	182

8	Green's Functions	187
8.1	Introduction	187
8.2	General Formulation	188
8.3	Poisson's Equation	189
8.4	Green's Function in One Dimension	190
8.5	Vector Potential	200
8.6	Summary	200
	Exercises	201
9	Laplace's Equation	205
9.1	Introduction	205
9.2	Forms of Laplace's Equation	206
9.3	Rectangular Coordinates	207
	9.3.1 Eigenvalue Problems	207
9.4	Cylindrical Coordinates	211
9.5	Spherical Coordinates	214
9.6	Summary	219
10	Time Dependence	221
10.1	Introduction	221
10.2	Faraday's Law	221
10.3	Displacement Current	224
10.4	Magnetostatic Energy	226
10.5	Maxwell's Equations	228
10.6	Summary	230
	Exercises	230
11	Electromagnetic Waves	239
11.1	Introduction	239
11.2	Wave Equations	240
11.3	Plane Waves	241
11.4	Plane Wave Structure	244
	11.4.1 Polarization	246
11.5	General Wave Solutions	249
	11.5.1 Spread of Waves	249
	11.5.2 Representation in Plane Waves	250
	11.5.3 Fourier Transform	250
11.6	Fourier Transformed Equations	254
11.7	Scalar and Vector Potentials	255
11.8	Summary	256
	Exercises	257
12	Energy and Momentum	261
12.1	Introduction	261
12.2	Transport Theorem	262
12.3	Electromagnetic Field Energy	263
12.4	Electromagnetic Field Momentum	266

12.5	Static Field Energies	269
12.6	Summary	270
	Exercises	270
13	Special Relativity	273
13.1	Introduction	273
13.2	The New Kinematics	274
13.2.1	Time	275
13.2.2	Space	278
13.2.3	Lorentz Transformation	280
13.3	Minkowski Space-Time	280
13.3.1	Four Dimensions	280
13.3.2	Four Vectors	282
13.3.3	The Minkowski Axiom	282
13.3.4	The Light Cone	283
13.4	Formal Lorentz Transform	284
13.5	Time and Space	285
13.5.1	Time Dilation	286
13.5.2	Space Contraction	287
13.5.3	Velocities	289
13.6	Tensors	291
13.7	Metric Space	293
13.8	Four-Velocity	294
13.9	Mass, Momentum, and Energy	295
13.9.1	Mass	295
13.9.2	Four-Momentum	296
13.9.3	Energy	297
13.10	Electrodynamics	300
13.10.1	Field Equations	300
13.10.2	Derivatives	300
13.10.3	Current and Potential Vectors	302
13.10.4	Electrodynamic Covariance	303
13.10.5	Field Strength Tensor	304
13.11	Moving Charges	306
13.12	Summary	311
	Exercises	312
14	Radiation	317
14.1	Introduction	317
14.2	Waves from Sources	317
14.3	Liénard–Wiechert Potentials	322
14.4	Plane Waves	326
14.5	Sources	327
14.5.1	Dipole Radiation	328
14.5.2	Charge in a Magnetic Field	330
14.6	Summary	331
	Exercises	332

15	Fields in Matter	335
15.1	Introduction	335
15.2	Experiments	336
15.2.1	Dielectrics in Capacitors	336
15.2.2	Solid Dielectrics	337
15.2.3	Magnetic Cores in Inductors	339
15.2.4	Magnetism in Solids	340
15.3	Potentials from Slowly Varying Fields	342
15.3.1	Atoms and Multipole Expansions	342
15.3.2	Polarization and Magnetization Densities	345
15.3.3	Polarization Charges and Magnetization Currents	347
15.4	Interaction of Fields	349
15.5	Maxwell's Equations in Matter	350
15.6	Constitutive Equations	351
15.6.1	Polarization	351
15.6.2	Magnetization	351
15.6.3	Permittivity and Permeability	352
15.7	Boundary Conditions on Fields	352
15.7.1	Electric Field	353
15.7.2	Magnetic Field	355
15.8	Ferromagnetism	357
15.8.1	Hysteresis	360
15.8.2	Modern Directions	361
15.9	Summary	361
	Exercises	362
16	Waves in Dispersive Media	373
16.1	Introduction	373
16.2	Waves in Matter	374
16.2.1	Representation of Waves	374
16.2.2	Dispersion Relation in Matter	375
16.2.3	Transverse and Longitudinal Waves	376
16.2.4	Wave Conductivity	377
16.2.5	Wave Energy	377
16.3	Nearly Monochromatic Waves	378
16.3.1	Dispersion of Monochromatic Waves	378
16.3.2	Time and Space Averages	379
16.3.3	Field Energy	381
16.3.4	Particle Energy	385
16.4	Note on Group Velocity	387
16.5	Application	389
16.6	Summary	391
	Exercises	392

Appendix A Vector Calculus	393
A.1 Differential Operators.....	393
A.2 Differential Operator Identities.....	394
Appendix B Dirac Delta Sequences	395
Appendix C Divergence and Curl of B	397
Appendix D Green's Theorem	399
Appendix E Laplace's Equation	401
Appendix F Poisson's Equation	407
Appendix G Helmholtz' Equation	409
Appendix H Legendre's Equation	413
Appendix I Jacobians	417
Appendix J Dispersion	423
Appendix K Answers to Selected Exercises	429
References	435
Index	439