

# Table of Contents

|          |                                         |    |
|----------|-----------------------------------------|----|
| <b>1</b> | <b>Introduction</b>                     | 1  |
| 1.1      | Enumeration                             | 2  |
| 1.2      | Running Time of Algorithms              | 5  |
| 1.3      | Linear Optimization Problems            | 8  |
| 1.4      | Sorting                                 | 9  |
|          | Exercises                               | 11 |
|          | References                              | 12 |
| <b>2</b> | <b>Graphs</b>                           | 13 |
| 2.1      | Basic Definitions                       | 13 |
| 2.2      | Trees, Circuits, and Cuts               | 17 |
| 2.3      | Connectivity                            | 24 |
| 2.4      | Eulerian and Bipartite Graphs           | 31 |
| 2.5      | Planarity                               | 34 |
| 2.6      | Planar Duality                          | 41 |
|          | Exercises                               | 43 |
|          | References                              | 47 |
| <b>3</b> | <b>Linear Programming</b>               | 51 |
| 3.1      | Polyhedra                               | 52 |
| 3.2      | The Simplex Algorithm                   | 56 |
| 3.3      | Implementation of the Simplex Algorithm | 60 |
| 3.4      | Duality                                 | 63 |
| 3.5      | Convex Hulls and Polytopes              | 67 |
|          | Exercises                               | 68 |
|          | References                              | 70 |
| <b>4</b> | <b>Linear Programming Algorithms</b>    | 73 |
| 4.1      | Size of Vertices and Faces              | 73 |
| 4.2      | Continued Fractions                     | 76 |
| 4.3      | Gaussian Elimination                    | 79 |
| 4.4      | The Ellipsoid Method                    | 82 |
| 4.5      | Khachiyan's Theorem                     | 88 |
| 4.6      | Separation and Optimization             | 90 |
|          | Exercises                               | 97 |
|          | References                              | 99 |

- 5 Integer Programming . . . . . 101**
  - 5.1 The Integer Hull of a Polyhedron . . . . . 103
  - 5.2 Unimodular Transformations . . . . . 107
  - 5.3 Total Dual Integrality . . . . . 109
  - 5.4 Totally Unimodular Matrices . . . . . 112
  - 5.5 Cutting Planes . . . . . 117
  - 5.6 Lagrangean Relaxation . . . . . 121
  - Exercises . . . . . 123
  - References . . . . . 127
  
- 6 Spanning Trees and Arborescences . . . . . 131**
  - 6.1 Minimum Spanning Trees . . . . . 131
  - 6.2 Minimum Weight Arborescences . . . . . 138
  - 6.3 Polyhedral Descriptions . . . . . 142
  - 6.4 Packing Spanning Trees and Arborescences . . . . . 145
  - Exercises . . . . . 148
  - References . . . . . 153
  
- 7 Shortest Paths . . . . . 157**
  - 7.1 Shortest Paths From One Source . . . . . 158
  - 7.2 Shortest Paths Between All Pairs of Vertices . . . . . 162
  - 7.3 Minimum Mean Cycles . . . . . 165
  - Exercises . . . . . 167
  - References . . . . . 169
  
- 8 Network Flows . . . . . 173**
  - 8.1 Max-Flow-Min-Cut Theorem . . . . . 174
  - 8.2 Menger's Theorem . . . . . 178
  - 8.3 The Edmonds-Karp Algorithm . . . . . 180
  - 8.4 Dinic's, Karzanov's, and Fujishige's Algorithm . . . . . 182
  - 8.5 The Goldberg-Tarjan Algorithm . . . . . 186
  - 8.6 Gomory-Hu Trees . . . . . 190
  - 8.7 The Minimum Capacity of a Cut in an Undirected Graph . . . . . 196
  - Exercises . . . . . 199
  - References . . . . . 205
  
- 9 Minimum Cost Flows . . . . . 211**
  - 9.1 Problem Formulation . . . . . 211
  - 9.2 An Optimality Criterion . . . . . 214
  - 9.3 Minimum Mean Cycle-Cancelling Algorithm . . . . . 216
  - 9.4 Successive Shortest Path Algorithm . . . . . 219
  - 9.5 Orlin's Algorithm . . . . . 223
  - 9.6 The Network Simplex Algorithm . . . . . 227
  - 9.7 Flows Over Time . . . . . 231
  - Exercises . . . . . 233
  - References . . . . . 237

|           |                                                          |     |
|-----------|----------------------------------------------------------|-----|
| <b>10</b> | <b>Maximum Matchings</b>                                 | 241 |
| 10.1      | Bipartite Matching                                       | 242 |
| 10.2      | The Tutte Matrix                                         | 244 |
| 10.3      | Tutte's Theorem                                          | 246 |
| 10.4      | Ear-Decompositions of Factor-Critical Graphs             | 249 |
| 10.5      | Edmonds' Matching Algorithm                              | 255 |
|           | Exercises                                                | 264 |
|           | References                                               | 268 |
| <b>11</b> | <b>Weighted Matching</b>                                 | 273 |
| 11.1      | The Assignment Problem                                   | 274 |
| 11.2      | Outline of the Weighted Matching Algorithm               | 276 |
| 11.3      | Implementation of the Weighted Matching Algorithm        | 279 |
| 11.4      | Postoptimality                                           | 292 |
| 11.5      | The Matching Polytope                                    | 293 |
|           | Exercises                                                | 296 |
|           | References                                               | 298 |
| <b>12</b> | <b><math>b</math>-Matchings and <math>T</math>-Joins</b> | 301 |
| 12.1      | $b$ -Matchings                                           | 301 |
| 12.2      | Minimum Weight $T$ -Joins                                | 305 |
| 12.3      | $T$ -Joins and $T$ -Cuts                                 | 309 |
| 12.4      | The Padberg-Rao Theorem                                  | 313 |
|           | Exercises                                                | 315 |
|           | References                                               | 318 |
| <b>13</b> | <b>Matroids</b>                                          | 321 |
| 13.1      | Independence Systems and Matroids                        | 321 |
| 13.2      | Other Matroid Axioms                                     | 325 |
| 13.3      | Duality                                                  | 329 |
| 13.4      | The Greedy Algorithm                                     | 333 |
| 13.5      | Matroid Intersection                                     | 338 |
| 13.6      | Matroid Partitioning                                     | 343 |
| 13.7      | Weighted Matroid Intersection                            | 345 |
|           | Exercises                                                | 349 |
|           | References                                               | 351 |
| <b>14</b> | <b>Generalizations of Matroids</b>                       | 355 |
| 14.1      | Greedoids                                                | 355 |
| 14.2      | Polymatroids                                             | 359 |
| 14.3      | Minimizing Submodular Functions                          | 363 |
| 14.4      | Schrijver's Algorithm                                    | 365 |
| 14.5      | Symmetric Submodular Functions                           | 369 |
|           | Exercises                                                | 371 |
|           | References                                               | 374 |

|           |                                                     |     |
|-----------|-----------------------------------------------------|-----|
| <b>15</b> | <b><i>NP</i>-Completeness</b>                       | 377 |
| 15.1      | Turing Machines                                     | 377 |
| 15.2      | Church's Thesis                                     | 380 |
| 15.3      | <i>P</i> and <i>NP</i>                              | 385 |
| 15.4      | Cook's Theorem                                      | 389 |
| 15.5      | Some Basic <i>NP</i> -Complete Problems             | 392 |
| 15.6      | The Class <i>coNP</i>                               | 400 |
| 15.7      | <i>NP</i> -Hard Problems                            | 402 |
|           | Exercises                                           | 406 |
|           | References                                          | 410 |
| <b>16</b> | <b>Approximation Algorithms</b>                     | 413 |
| 16.1      | Set Covering                                        | 414 |
| 16.2      | The Max-Cut Problem                                 | 419 |
| 16.3      | Colouring                                           | 425 |
| 16.4      | Approximation Schemes                               | 433 |
| 16.5      | Maximum Satisfiability                              | 435 |
| 16.6      | The <i>PCP</i> Theorem                              | 440 |
| 16.7      | L-Reductions                                        | 444 |
|           | Exercises                                           | 450 |
|           | References                                          | 454 |
| <b>17</b> | <b>The Knapsack Problem</b>                         | 459 |
| 17.1      | Fractional Knapsack and Weighted Median Problem     | 459 |
| 17.2      | A Pseudopolynomial Algorithm                        | 462 |
| 17.3      | A Fully Polynomial Approximation Scheme             | 464 |
| 17.4      | Multi-Dimensional Knapsack                          | 467 |
|           | Exercises                                           | 468 |
|           | References                                          | 469 |
| <b>18</b> | <b>Bin-Packing</b>                                  | 471 |
| 18.1      | Greedy Heuristics                                   | 471 |
| 18.2      | An Asymptotic Approximation Scheme                  | 477 |
| 18.3      | The Karmarkar-Karp Algorithm                        | 481 |
|           | Exercises                                           | 484 |
|           | References                                          | 486 |
| <b>19</b> | <b>Multicommodity Flows and Edge-Disjoint Paths</b> | 489 |
| 19.1      | Multicommodity Flows                                | 490 |
| 19.2      | Algorithms for Multicommodity Flows                 | 494 |
| 19.3      | Sparsest Cut and Max-Flow Min-Cut Ratio             | 499 |
| 19.4      | The Leighton-Rao Theorem                            | 500 |
| 19.5      | Directed Edge-Disjoint Paths Problem                | 503 |
| 19.6      | Undirected Edge-Disjoint Paths Problem              | 507 |
|           | Exercises                                           | 513 |
|           | References                                          | 517 |

|           |                                             |     |
|-----------|---------------------------------------------|-----|
| <b>20</b> | <b>Network Design Problems</b>              | 521 |
| 20.1      | Steiner Trees                               | 522 |
| 20.2      | The Robins-Zelikovsky Algorithm             | 527 |
| 20.3      | Survivable Network Design                   | 532 |
| 20.4      | A Primal-Dual Approximation Algorithm       | 536 |
| 20.5      | Jain's Algorithm                            | 544 |
|           | Exercises                                   | 550 |
|           | References                                  | 553 |
| <b>21</b> | <b>The Traveling Salesman Problem</b>       | 557 |
| 21.1      | Approximation Algorithms for the TSP        | 557 |
| 21.2      | Euclidean TSP                               | 562 |
| 21.3      | Local Search                                | 569 |
| 21.4      | The Traveling Salesman Polytope             | 575 |
| 21.5      | Lower Bounds                                | 581 |
| 21.6      | Branch-and-Bound                            | 584 |
|           | Exercises                                   | 586 |
|           | References                                  | 589 |
| <b>22</b> | <b>Facility Location</b>                    | 593 |
| 22.1      | The Uncapacitated Facility Location Problem | 593 |
| 22.2      | Rounding Linear Programming Solutions       | 595 |
| 22.3      | Primal-Dual Algorithms                      | 597 |
| 22.4      | Scaling and Greedy Augmentation             | 603 |
| 22.5      | Bounding the Number of Facilities           | 606 |
| 22.6      | Local Search                                | 609 |
| 22.7      | Capacitated Facility Location Problems      | 615 |
| 22.8      | Universal Facility Location                 | 618 |
|           | Exercises                                   | 624 |
|           | References                                  | 626 |
|           | <b>Notation Index</b>                       | 629 |
|           | <b>Author Index</b>                         | 633 |
|           | <b>Subject Index</b>                        | 645 |