

# Contents

## Preface — VII

|          |                                                 |
|----------|-------------------------------------------------|
| <b>1</b> | <b>Interacting particle systems — 1</b>         |
| 1.1      | General framework — 3                           |
| 1.1.1    | Basic definitions — 4                           |
| 1.1.2    | The network of interactions — 5                 |
| 1.1.3    | The local transition rates — 9                  |
| 1.1.4    | Existence on infinite graphs — 13               |
| 1.2      | The contact process (invasion) — 19             |
| 1.2.1    | Monotonicity and critical values — 21           |
| 1.2.2    | Attractiveness and self-duality — 24            |
| 1.2.3    | The process on integer lattices — 27            |
| 1.2.4    | The process on a finite set — 35                |
| 1.2.5    | The process on homogeneous trees — 40           |
| 1.3      | The voter model (competition) — 45              |
| 1.3.1    | Duality, clustering, and coexistence — 47       |
| 1.3.2    | Cluster size (spatial correlations) — 50        |
| 1.3.3    | Occupation time (temporal correlations) — 53    |
| 1.3.4    | Voter model consensus time — 56                 |
| 1.3.5    | The Richardson growth model — 57                |
| 1.3.6    | The biased voter model — 59                     |
| 1.4      | The multitype contact process — 63              |
| 1.4.1    | Duality, clustering, and coexistence — 65       |
| 1.4.2    | The biased multitype contact process — 69       |
| 1.4.3    | Multitype contact process on trees — 72         |
| <br>     |                                                 |
| <b>2</b> | <b>Epidemiology: disease dynamics — 75</b>      |
| 2.1      | Spatial SIR and SEIS models — 79                |
| 2.1.1    | Spatial epidemic with removal — 79              |
| 2.1.2    | Spatial epidemic with recovery — 83             |
| 2.1.3    | Partial immunization process — 87               |
| 2.1.4    | Contact process with a latent state — 90        |
| 2.1.5    | Contact process with an asymptomatic state — 94 |
| 2.2      | Stacked contact processes — 97                  |
| 2.2.1    | The basic spatial epidemic — 99                 |
| 2.2.2    | Models with vertical transmission — 100         |
| 2.2.3    | Partial vertical transmission — 106             |
| 2.3      | Multistrain dynamics — 108                      |
| 2.3.1    | Drug-resistant diseases — 109                   |
| 2.3.2    | Contact process with mutations — 110            |

- 2.3.3 The process on trees — 112
- 2.4 The role of social behavior — 113
- 2.4.1 Risky behavior and sexually transmitted diseases — 115
- 2.4.2 HIV dynamics — 117
- 2.4.3 Social clusters with collective recoveries — 118
- 2.4.4 Social clusters with individual recoveries — 120
- 2.5 Epidemics on social networks — 122
- 2.5.1 Random and heterogeneous trees (computer networks) — 123
- 2.5.2 Finite random graphs (social networks) — 127
- 2.5.3 Small-world networks — 129
- 2.6 Epidemics in dynamic environments — 132
- 2.6.1 Dynamic vertices — 132
- 2.6.2 Dynamic edges — 134
- 2.6.3 Evolving scale-free networks — 136
- 2.6.4 The partner model — 136

### **3 Population ecology: single-species dynamics — 139**

- 3.1 Biased annihilating branching process — 142
- 3.1.1 The symmetric case — 143
- 3.1.2 The biased process — 145
- 3.1.3 Adding natural deaths — 147
- 3.2 Sexual reproduction — 147
- 3.2.1 Threshold models — 149
- 3.2.2 Rapid stirring — 152
- 3.2.3 Long range interactions — 154
- 3.2.4 Diploid dioecious species — 157
- 3.3 Metapopulation models — 157
- 3.3.1 The two-stage contact process — 158
- 3.3.2 Spatially implicit colonies — 165
- 3.3.3 Colonies with Allee effect — 169
- 3.4 Models with mass extinctions — 174
- 3.4.1 The two-scale contact process — 175
- 3.4.2 The role of aggregation — 180
- 3.4.3 Colonization and collapse — 182
- 3.4.4 Tornadoes versus forest fires — 184
- 3.5 Contact process in random environments — 187
- 3.5.1 One-dimensional random environment — 188
- 3.5.2 Multidimensional random environment — 192
- 3.5.3 Heterogeneous environment — 195
- 3.5.4 Sterile insect control strategy — 197

|          |                                                         |
|----------|---------------------------------------------------------|
| <b>4</b> | <b>Community ecology: multispecies dynamics — 200</b>   |
| 4.1      | Competition models — 203                                |
| 4.1.1    | The Lotka–Volterra model — 203                          |
| 4.1.2    | Spatial and temporal heterogeneities — 210              |
| 4.1.3    | More coexistence results — 214                          |
| 4.2      | Other types of biological interactions — 218            |
| 4.2.1    | The static-host model (commensalism) — 219              |
| 4.2.2    | The dynamic-host model (parasitism and mutualism) — 223 |
| 4.2.3    | The symbiotic contact process (cooperation) — 231       |
| 4.2.4    | Allelopathic interactions (amensalism) — 233            |
| 4.2.5    | Predator–prey systems (predation) — 241                 |
| 4.3      | Ecological succession — 245                             |
| 4.3.1    | Grass–bushes–trees model — 245                          |
| 4.3.2    | Brackens–beeches successional model — 250               |
| 4.3.3    | Cyclic particle systems — 254                           |
| 4.4      | Mutual unbounded growth — 260                           |
| 4.4.1    | Multitype Richardson model — 260                        |
| 4.4.2    | Biased process and other growth models — 264            |
| 4.4.3    | The chase–escape model — 267                            |
| <b>5</b> | <b>Population genetics: gene dynamics — 270</b>         |
| 5.1      | The stepping-stone model — 274                          |
| 5.1.1    | Asymptotics for the size of a family — 275              |
| 5.1.2    | Identity by descent and genealogy of a sample — 278     |
| 5.1.3    | Two-allele model with selection — 283                   |
| 5.1.4    | The role of extinction and recolonization — 289         |
| 5.2      | The multitype voter model with mutations — 291          |
| 5.2.1    | The process with no mutations — 292                     |
| 5.2.2    | Species–area curves — 293                               |
| 5.2.3    | The abundance of species — 296                          |
| 5.3      | Spatial diploid models — 296                            |
| 5.3.1    | Non-Mendelian model — 297                               |
| 5.3.2    | Sickle-cell disease — 301                               |
| 5.4      | Hybrid zones — 304                                      |
| 5.4.1    | Number of inversions and tightness — 305                |
| 5.4.2    | One-dimensional competition models — 312                |
| 5.4.3    | Heterogeneous environments — 315                        |
| <b>6</b> | <b>Sociology: opinion dynamics — 320</b>                |
| 6.1      | Threshold voter model and other variants — 324          |
| 6.1.1    | Coexistence ( $\theta < 1/4$ ) — 326                    |
| 6.1.2    | Clustering ( $1/4 < \theta < 1/2$ ) — 329               |

- 6.1.3 Fixation ( $\theta > 1/2$ ) — **332**
- 6.1.4 Galam's majority rule — **334**
- 6.1.5 Latent and zealot voter models — **340**
- 6.2 The Axelrod model — **342**
- 6.2.1 Connection with annihilating-coalescing particles — **345**
- 6.2.2 Fluctuation/consensus in dimension one — **346**
- 6.2.3 Fixation/discordance in dimension one — **348**
- 6.2.4 Natural variants — **350**
- 6.3 Opinion graphs and confidence threshold — **352**
- 6.3.1 Imitation rule on the integers — **354**
- 6.3.2 The vectorial Deffuant model — **357**
- 6.3.3 Imitation and attraction rules on finite graphs — **359**
- 6.4 Continuous opinion spaces — **362**
- 6.4.1 The imitation rule — **363**
- 6.4.2 The averaging process — **365**
- 6.4.3 The Deffuant model — **367**
- 6.4.4 Mutual and unilateral compromise on finite graphs — **375**
- 6.5 Adaptive voter models — **377**
- 6.5.1 Main conjectures — **378**
- 6.5.2 Approximation of the critical value  $r_c$  — **382**
- 6.5.3 Rigorous results on dense graphs — **384**

## **7 Spatial games: strategy dynamics — 387**

- 7.1 Symmetric two-person games — **391**
- 7.1.1 Popular games with two strategies — **392**
- 7.1.2 Replicator dynamics — **395**
- 7.2 Cooperation versus defection — **398**
- 7.2.1 Benefit-to-cost  $b/c$  rules — **398**
- 7.2.2 Rigorous results when  $w \ll 1$  — **400**
- 7.2.3 Cooperation and kin recognition — **403**
- 7.3 More games (weak selection) — **407**
- 7.3.1 Birth-death updating process — **407**
- 7.3.2 Death-birth updating process — **410**
- 7.3.3 Three-strategy games — **413**
- 7.3.4 Effect of the selection parameter — **415**
- 7.4 More updating rules (strong selection) — **417**
- 7.4.1 Generalized birth-death process — **418**
- 7.4.2 Death-birth updating process — **423**
- 7.4.3 Death-birth of the fittest — **427**
- 7.4.4 Best-response dynamics — **431**
- 7.5 Games in the life and social sciences — **433**
- 7.5.1 Cancer modeling — **434**

- 7.5.2      Producer–consumer system — **437**
- 7.5.3      Econophysics: money dynamics — **441**

**Bibliography — 447**

**Index — 463**