

Miguel Orszag

Quantum Optics

Including Noise Reduction,
Trapped Ions, Quantum Trajectories,
and Decoherence

With 75 Figures
and 79 Problems with Hints for Solutions



Springer

Contents

1. Einstein's Theory of Atom-Radiation Interaction	1
1.1 The A and B Coefficients	1
1.2 Thermal Equilibrium	3
1.3 Photon Distribution and Fluctuations	3
1.4 Light Beam Incident on Atoms	4
1.5 Elementary Laser Theory	5
1.5.1 Threshold and Population Inversion	6
1.5.2 Steady State	6
1.5.3 Linear Stability Analysis	7
Problems	9
2. Atom-Field Interaction: Semiclassical Approach	11
2.1 Broad-Band Radiation Spectrum	14
2.2 Rabi Oscillations	15
2.3 Bloch's Equations	16
2.4 Decay to an Unobserved Level	17
2.5 Decay Between Levels	18
2.6 Optical Nutation	18
Problems	19
3. Quantization of the Electromagnetic Field	21
3.1 Fock States	24
3.2 Density of Modes	25
3.3 Commutation Relations	26
Problems	27
4. States of the Electromagnetic Field I	29
4.1 Further Properties	30
4.1.1 Coherent States are Minimum Uncertainty States	30
4.1.2 Coherent States are not Orthogonal	30
4.1.3 Coherent States are Overcomplete	31
4.1.4 The Displacement Operator	31
4.1.5 Photon Statistics	32
4.1.6 Coordinate Representation	33

4.2	Mixed State. Thermal Radiation	33
	Problems	35
5.	States of the Electromagnetic Field II	39
5.1	Squeezed States. General Properties and Detection	39
5.1.1	The Squeeze Operator and the Squeezed State	41
5.1.2	The Squeezed State is an Eigenstate of A	42
5.1.3	Calculation of Moments with Squeezed States	42
5.1.4	Quadrature Fluctuations	43
5.1.5	Photon Statistics	44
5.2	Multimode Squeezed States	44
5.3	Detection of Squeezed States	45
5.3.1	Ordinary Homodyne Detection	46
5.3.2	Balanced Homodyne Detection	47
5.3.3	Heterodyne Detection	48
	Problems	48
6.	Quantum Theory of Coherence	51
6.1	One-Atom Detector	52
6.2	The n -Atom Detector	55
6.3	General Properties of the Correlation Functions	56
6.4	Young's Interference and First-Order Correlation	58
6.5	Second-Order Correlations. Photon Bunching and Antibunching	60
6.5.1	Classical Second-Order Coherence	60
6.5.2	Quantum Theory of Second-Order Coherence	62
6.6	Photon Counting	65
6.6.1	Some Simple Examples	68
6.6.2	Quantum Mechanical Photon Count Distribution	68
6.6.3	Particular Examples	69
	Problems	70
7.	Phase Space Description	71
7.1	Q -Representation. Antinormal Ordering	71
7.1.1	Normalization	71
7.1.2	Average of Antinormally Ordered Products	72
7.1.3	Some Examples	72
7.1.4	The Density Operator in Terms of the Function Q	73
7.2	Characteristic Function	74
7.3	P -Representation: Normal Ordering	74
7.3.1	Normalization	74
7.3.2	Averages of Normally Ordered Products	75
7.3.3	Some Interesting Properties	75
7.3.4	Some Examples	75
7.4	The Wigner Distribution: Symmetric Ordering	77

7.4.1 Moments	77
Problems	78
8. Atom-Field Interaction	81
8.1 Atom-Field Hamiltonian and the Dipole Approximation	81
8.2 A Two-Level Atom Interacting with a Single Field Mode	83
8.3 The Dressed State Picture. Quantum Rabi Oscillations	85
8.4 Collapse and Revivals	88
Problems	92
9. System-Reservoir Interactions	93
9.1 Quantum Theory of Damping	93
9.2 General Properties	96
9.3 Expectation Values of Relevant Physical Quantities	97
9.4 Time Evolution of the Density Matrix Elements	98
9.5 The Glauber-Sudarshan Representation and the Fokker-Planck Equation	100
9.6 Time-Dependent Solution. The Method of the Eigenfunctions	101
9.6.1 General Solution	103
9.7 Langevin's Equations	103
9.7.1 Calculation of the Correlation Function $\langle F(t')F(t'')^\dagger \rangle_B$	105
9.7.2 Differential Equation for the Photon Number	105
9.8 Other Master Equations	105
9.8.1 Two-Level Atom in a Thermal Bath	105
9.8.2 Damped Harmonic Oscillator in a Squeezed Bath	107
9.8.3 Application. Spontaneous Decay in a Squeezed Vacuum	109
Problems	110
10. Resonance Fluorescence	113
10.1 Background	113
10.2 Heisenberg's Equations	114
10.3 Spectral Density and the Wiener-Khinchine Theorem	118
10.4 Emission Spectra from Strongly Driven Two-Level Atoms	120
10.5 Intensity Correlations	123
Problems	127
11. Quantum Laser Theory. Master Equation Approach	129
11.1 Heuristic Discussion of Injection Statistics	131
11.2 Master Equation for Generalized Pump Statistics	131
11.3 The Quantum Theory of the Laser. Random Injection ($p = 0$)	133
11.3.1 Photon Statistics	134
11.3.2 The Fokker-Planck Equation. Laser Linewidth	137
11.3.3 Alternative Derivation of the Laser Linewidth	138

11.4 Quantum Theory of the Micromaser.	
Random Injection ($p = 0$)	140
11.4.1 The Micromaser	141
11.4.2 Trapping States	144
11.5 Quantum Theory of the Laser and the Micromaser	
with Pump Statistics ($p \neq 0$)	146
Problems	150
12. Quantum Laser Theory. Langevin Approach	151
12.1 Quantum Langevin Equations	151
12.1.1 Generalized Einstein Relations	152
12.1.2 Atomic Noise Moments	153
12.2 C -Number Langevin Equations	157
12.2.1 Adiabatic Approximation	158
12.3 Phase and Intensity Fluctuations	159
12.4 Discussion	160
Problems	162
13. Quantum Noise Reduction I	163
13.1 Correlated Emission Laser (CEL) Systems:	
The Quantum Beat Laser	164
13.1.1 The Model	164
13.1.2 The Solution	167
13.1.3 The Master Equation	167
13.1.4 Photon Statistics	169
13.2 Other CEL Systems	171
Problems	172
14. Quantum Noise Reduction II	173
14.1 Introduction to Non-linear Optics	173
14.1.1 Multiple Photon Transitions	173
14.2 Parametric Processes Without Losses	177
14.3 Input-Output Theory	179
14.4 The Degenerate Parametric Oscillator	183
14.5 Experimental Results	185
Problems	187
15. Quantum Phase	191
15.1 The Dirac Phase	191
15.2 The Louisell Phase	192
15.3 The Susskind-Glogower Phase	192
15.4 The Pegg-Barnett Phase	195
15.4.1 Applications	198
15.5 Phase Fluctuations in a Laser	200
Problems	203

16. Quantum Trajectories	205
16.1 Monte Carlo Wavefunction Method	206
16.1.1 The Monte Carlo Method is Equivalent, on Average, to the Master Equation	207
16.2 The Stochastic Schrödinger Equation	209
16.3 Stochastic Schrödinger Equations and Dissipative Systems ...	211
16.4 Simulation of a Monte Carlo SSE	213
16.5 Simulation of the Homodyne SSDE	218
16.6 Numerical Results and Localization	221
16.6.1 Evolution of Quantum Jumps	222
16.6.2 Diffusion-like Evolution	224
16.6.3 Analytical Proof of Localization	225
16.7 Conclusions	228
Problems	229
17. Atom Optics	231
17.1 Optical Elements	231
17.2 Atomic Diffraction from an Optical Standing Wave	232
17.2.1 Theory	233
17.2.2 Particular Cases	236
17.3 Atomic Focusing	239
17.3.1 The Model	240
17.3.2 Initial Conditions and Solution	241
17.3.3 Quantum and Classical Foci	242
17.3.4 Thin versus Thick Lenses	243
17.3.5 The Quantum Focal Curve	244
17.3.6 Aberrations	245
Problems	246
18. Measurements, Quantum Limits and all That	247
18.1 Quantum Standard Limit	247
18.1.1 Quantum Standard Limit for a Free Particle	247
18.1.2 Standard Quantum Limit for an Oscillator	248
18.1.3 Thermal Effects	249
18.2 Quantum Non-demolition (QND) Measurements	250
18.2.1 The Free System	250
18.2.2 Monitoring a Classical Force	252
18.2.3 Effect of the Measuring Apparatus or Probe	253
18.3 QND Measurement of the Number of Photons in a Cavity ...	253
18.3.1 The Model	253
18.3.2 The System-Probe Interaction	255
18.3.3 Measuring the Atomic Phase with Ramsey Fields	255
18.3.4 QND Measurement of the Photon Number	259
18.4 Quantum Theory of Continuous Photodetection Processes ...	261

18.4.1 Continuous Measurement in a Two-Mode System.	
Phase Narrowing	263
Problems	267
19. Trapped Ions	269
19.1 Paul Trap	269
19.1.1 Stability Analysis	272
19.2 The Trapped Ion in the Raman Scheme	276
19.2.1 The Model and the Effective Hamiltonian	276
19.2.2 The Lamb-Dicke Expansion and Raman Cooling	281
19.2.3 The Dynamical Evolution	282
19.2.4 QND Measurements of Vibrational States	285
19.2.5 Generation of Non-classical Vibrational States	288
Problems	289
20. Decoherence	291
20.1 Dynamics of the Correlations	293
20.2 How Long Does it Take to Decohere?	295
Problems	299
A. Operator Relations	301
A.1 Theorem 1	301
A.2 Theorem 2. The Baker-Campbell-Hausdorff Relation	302
A.3 Theorem 3. Similarity Transformation	303
B. The Method of Characteristics	305
C. Proof of Eq. (12.37)	309
D. Stochastic Processes in a Nutshell	311
D.1 Introduction	311
D.2 Probability Concepts	312
D.3 Stochastic Processes	313
D.3.1 The Chapman-Kolmogorov Equation	313
D.4 The Fokker-Planck Equation	316
D.4.1 The Wiener Process	317
D.4.2 General Properties of the Fokker-Planck Equation	319
D.4.3 Steady State Solution	319
D.5 Stochastic Differential Equations	320
D.5.1 Ito versus Stratonovich Calculus	322
D.5.2 Ito's Formula	324
D.6 Approximate Methods	325
E. Derivation of the Homodyne Stochastic	
 Schrödinger Differential Equation	329

F. Fluctuations	333
Hints for Solutions of Problems	335
References	340
Index	355