

Table of Contents

0. Overview and Mathematical Glossary	1
Main Concepts	2
Possible Limitations	8
A Practical Application: The Aleks System	9
Potential Applications to Other Fields	10
On the Content and Organization of this Book	11
Glossary of Mathematical Concepts	12
Original Sources and Related Works	16
1. Knowledge Structures and Spaces	18
Fundamental Concepts	18
Knowledge Spaces	21
Closure Spaces	23
Substructures	25
Bases and Atoms	26
An Algorithm for Constructing the Base	29
An Algorithm for Generating a Space from its Base	30
Bases and Atoms: the Infinite Case*	33
The Surmise Relation	36
Quasi Ordinal Spaces	38
Original Sources and Related Works	40
Problems	41
2. Well-Graded Knowledge Structures	44
Essentially Finite Structures	44
A Well-Graded Family of Relations: the Biorders*	51
The Infinite Case*	55
Finite Learnability	58
Original Sources and Related Works	59
Problems	60

3. Surmise Systems	62
Basic Concepts	62
Knowledge Spaces and Surmise Systems	67
AND/OR Graphs	69
Surmise Functions and Wellgradedness	73
Hasse Systems	74
Resolubility and Acyclicity	79
Original Sources and Related Works	82
Problems	84
4. Skill Maps, Labels and Filters	86
Skills	86
Skill Maps: The Disjunctive Model	89
Minimal Skill Maps	91
Skill Maps: The Conjunctive Model	95
Skill Multimaps: The Competency Model	96
Labels and Filters	98
Original Sources and Related Works	102
Problems	102
5. Entailments and the Maximal Mesh	104
Entailments	105
Entail Relations	109
Meshability of Knowledge Structures	110
The Maximal Mesh	113
Original Sources and Related Works	117
Problems	118
6. Galois Connections*	120
Three Exemplary Correspondences	120
Closure Operators and Galois Connections	122
Lattices and Galois Connections	125
Knowledge Structures and Binary Relations	129
Granular Knowledge Structures and Granular Attributions	133
Knowledge Structures and Associations	137
Original Sources and Related Works	139
Problems	139

7. Probabilistic Knowledge Structures 142

 Basic Concepts and Examples142

 An Empirical Application146

 The Likelihood Ratio Procedure 151

 Learning Models155

 A Combinatorial Result158

 Markov Chain Models 160

 Probabilistic Substructures164

 Nomenclatures and Classifications167

 Independent Substructures168

 Original Sources and Related Works 172

 Problems 172

8. Stochastic Learning Paths 175

 A Knowledge Structure in Euclidean Geometry175

 Basic Concepts 177

 Basic Results 183

 Assumptions on Distributions186

 The Learning Latencies 187

 Empirical Predictions191

 Limitations of this Theory 194

 Simplifying Assumptions197

 Remarks on Application and Use of the Theory 198

 An Application of the Theory to the Case $n = 2$ 200

 Original Sources and Related Works 203

 Problems 204

9. Descriptive and Assessment Languages 206

 Languages and Decision Trees206

 Terminology210

 Recovering Ordinal Knowledge Structures213

 Recovering Knowledge Structures 216

 Original Sources and Related Works 218

 Problems 218

10. Uncovering the State of an Individual:

A Continuous Markov Procedure	219
<i>A Deterministic Algorithm</i>	219
Outline of a Markovian Stochastic Process	221
Notation and Basic Concepts	224
Special Cases	228
General Results	233
Uncovering the Latent State	235
Refining the Assessment	239
Proofs*	241
Original Sources and Related Works	245
Problems	246

11. Uncovering the State of an Individual:

A Markov Chain Procedure	247
Outline of a Markov Chain Procedure	247
The Stochastic Assessment Process	252
Combinatorial Assumptions on the Structure	254
Markov Chains Terminology	259
Results for the Fair Case	260
Uncovering a Stochastic State: Examples	264
Intractable Case	269
Original Sources and Related Works	271
Problems	272

12. Building the Knowledge Structure in Practice 274

Koppen's Algorithm for Constructing the Knowledge Space	275
Kambouri's Experiment	284
Results	290
Discussion of Kambouri's Results	298
Cosyn and Thiéry's Work	300
Refining a Knowledge Structure	304
Simulation of Various Refinements	306
Original Sources and Related Works	308
Problems	308

References.....310

Index of Names.....321

Index of Terms.....325