

Contents

1	Generating Equations of Non-Markov Theory	1
1.1	Generating Equations for Step-Driven Processes	1
1.1.1	Characteristic Function of Simple Relaxation Process	1
1.1.2	Derivation of the Generating Equation	3
1.1.3	Generating Equation Obtained from Time Reversibility	4
1.1.4	The First Generating Equation in the Quantum Case	5
1.1.5	Some Uses of Quantum Generating Equations	8
1.1.6	Second Quantum Generating Equation	9
1.1.7	Simplest Uses of the Last Generating Equation	12
1.2	Non-Markov Generating Equations. General Case	13
1.2.1	Derivation of Auxiliary Formula. Nonquantum Case	13
1.2.2	Characteristic Functional for Fluxes Under External Forces	14
1.2.3	Nonquantum Generating Equation	15
1.2.4	Consequence of the Generating Equation: the H-Theorem	19
1.2.5	Derivation of the Simplest FDRs from the Generating Equation	20
1.2.6	Quantum Case. Auxiliary Formulas	21
1.2.7	General Quantum Equation with an Arbitrary Operator Expression	22
1.2.8	Several Specific Generating Equations	24
1.2.9	Some Simple Uses of One of the Quantum Generating Equations	26
1.2.10	Quasiclassical Generating Equation in the Quantum Case	27
1.2.11	Derivatives of Quasiclassical Moments and Correlators	29
1.2.12	Quantum Markov Processes	33
1.3	Notes on References to Chapter 1	39
2	Nonequilibrium Thermodynamics of Open Systems	40
2.1	Open Systems. Examples	40
2.1.1	Open Systems and the Equations Describing Them	40
2.1.2	Inclusion of an Open System into a Closed System	41
2.1.3	Fluctuations in Open Systems with Relatively Small Nonlinearity	43

2.1.4	Principle of Minimal Free Energy Decrease of Minimal Entropy Production	44
2.1.5	Stability Criteria for Stationary States. Other Lyapunov Functions	49
2.1.6	Example of an Open Electrical System	51
2.1.7	Nonlinear Open System with Two Capacitances	52
2.1.8	Example of an Open System with Heat Exchange	54
2.1.9	A Simple Autocatalytic Reaction. Stability of the Stationary State	56
2.1.10	Admixture Flux Through a Substance	57
2.1.11	Admixture Flux with Nonlinear Diffusion	60
2.2	Some Generating Equations for Open Systems	62
2.2.1	The Markov Case	62
2.2.2	Generating Equation of the Non-Markov Theory for Systems Open in All Variables	64
2.3	H-Theorems and Relations for Nonequilibrium Stationary States . .	66
2.3.1	Quasi-Frenergy or Quasi-Entropy	66
2.3.2	Generating Equation	67
2.3.3	Conjugate of $\Psi(B)$	68
2.3.4	The H-Theorem	69
2.3.5	Another Form of the H-Theorem	71
2.3.6	One Consequence of the Last Theorem	73
2.3.7	Entropy as a Measure of Uncertainty of the Parameters B . .	75
2.3.8	Fluctuation-Dissipation Relations	77
2.3.9	FDRs in the Non-Markov Case	79
2.4	Methods of Calculating Correlators Near Nonequilibrium Kinetic Phase Transitions in the Markov Case	80
2.4.1	Equations for Derivatives of Quasi-Frenergy	80
2.4.2	Multistability. Nonequilibrium Kinetic Phase Transitions . .	83
2.4.3	Solution of the Equations for $\varphi_{\alpha\beta}, \psi_{\alpha\beta}$	86
2.4.4	Solution of Equations Determining Higher Derivatives	88
2.4.5	On the Accuracy of the Equations (2.3.90,91)	90
2.4.6	Two-Subscript Relations for Complex Variables	91
2.5	Example of Quasi-Frenergy Near a Nonequilibrium Phase Transi- tion. The Bénard Instability	93
2.5.1	Basic Equations	93
2.5.2	The Linearized Langevin Equation	94
2.5.3	Stationary Probability Density of Internal Parameters	95
2.5.4	A More General Expression for Quasi-Frenergy and Its Consequences	99
2.6	Fluctuations Near One-Component Nonequilibrium Kinetic Phase Transitions	102
2.6.1	One-Component Second-Order Transitions	102
2.6.2	Non-Gaussian Properties of Fluctuations Near a Second-Order Transition	103
2.6.3	One-Component First-Order Transition	105

2.6.4	Intermediate Case of Kinetic Phase Transition	107
2.6.5	Another Form of the First-Order Phase Transition	108
2.6.6	Example of a One-Component Transition: Tunnel Diode Circuit	110
2.6.7	Vibrational Two-Component Case Reducible to One-Component Case	113
2.6.8	Example of a Vibrational Phase Transition: Brusselator . . .	116
2.7	Parameter Fluctuations Near a Two-Component Kinetic Phase Transition	119
2.7.1	Determining the Matrix $\psi_{\alpha\beta}$	119
2.7.2	Calculation of Matrices of the Third Derivatives $\psi_{\alpha\beta\gamma}$. . .	121
2.7.3	Parameter Fluctuations in the Critical Region for Two-Component First-Order Transition	122
2.7.4	Two-Component Transition: A Special Case	124
2.7.5	Example of a Two-Component Kinetic Phase Transition . .	126
2.7.6	Parameter Fluctuations Given by $\psi_{\alpha\beta\gamma\delta}$	129
2.8	Notes on References to Chapter 2	133
3	The Kirchhoff's Form of Fluctuation-Dissipation Relations	134
3.1	Functions Describing Linear and Nonlinear Scattering, Reflection and Absorption of Waves	134
3.1.1	Incident and Transmitted Waves in One Particular Case . . .	134
3.1.2	Waves in Three-Dimensional Space	138
3.1.3	Introduction of Scattering Functions $U_{1,2,\dots,n}$	140
3.1.4	Scattering Functions and Absorption of Energy	141
3.1.5	Simplified Description of Wave Scattering	142
3.2	Linear and Quadratic FDRs (Generalized Kirchhoff's Laws)	143
3.2.1	Reciprocal Relation	143
3.2.2	The Linear FDR	146
3.2.3	Spectral Density of Emitted Waves. Conventional Form of Kirchhoff's Law	148
3.2.4	Three-Dimensional Form of Kirchhoff's Law	150
3.2.5	First Quadratic FDR	153
3.2.6	Second Quadratic FDR	155
3.2.7	Modified Stochastic Representation	156
3.2.8	Total (Unconditional) Correlators of Transmitted Waves . . .	157
3.3	Cubic Kirchhoff FDRs	158
3.3.1	Stochastic Representation of Transmitted Waves	158
3.3.2	Simplified Stochastic Representation	159
3.3.3	Relations for $U_{12,34}$	161
3.3.4	Relations for $U_{123,4}$	163
3.3.5	Relations for the Conditional Fourfold Correlator of Transmitted Waves	165
3.3.6	Basic Four-Subscript Relations in an Arbitrary Representation	166
3.4	Notes on References to Chapter 3	167

4	Method of Projection in Space of Phase Space Distributions and Irreversible Processes	168
4.1	Markov Processes in Microdynamics	168
4.1.1	Markov-Like Systems	168
4.1.2	The Operators $\hat{H}$ and $\hat{H}^-$	169
4.1.3	The Markov Operator $\hat{M}$	171
4.1.4	Properties of the Master Equation Operator	172
4.1.5	Transformation of the Markov Operator to Generalized Fokker-Planck Operator Form	174
4.1.6	Coefficients of the Master Equation $K_{\alpha_1 \dots \alpha_j}(A)$	176
4.1.7	The Zwanzig Equation	179
4.1.8	One Method of Using a Large Parameter to Derive the Markov Equation	181
4.1.9	Example of Derivation of the Master Equation	183
4.1.10	More Exact Formulas for the Markov Operator	192
4.2	Derivation of Linear FDR of the First Kind and Reciprocal Relation Using Projection Operator	194
4.2.1	Modified Projection Operator	194
4.2.2	Derivation of the Mori Formula	195
4.2.3	Linear FDR of the First Kind	197
4.2.4	Proof of Stationarity of Fluctuational Inputs $F_\alpha(t)$	198
4.2.5	Derivation of Reciprocal Relation for $\Phi_{1,2}$	199
	Appendices	202
A1.	Derivation of Equation (1.1.59)	202
A1.1	Identity for $\Phi(p_1, \dots, p_m)$	202
A1.2	Relation for Moments	203
A2.	Some Additional Results Concerning Nonlinear Fluctuation-Dissipation Relations	204
A2.1	The Nonquantum Cubic FDRs of the First Kind	204
A2.2	FDRs of the Second Kind and Transformation of the Stable Point	206
A2.3	Cubic FDRs of the Third Kind for $Q_{1,23} \neq 0$	215
A2.4	The Cubic Kirchhoff-Type Relations for $U_{1,23} \neq 0$	216
A3.	Linear Transformation of Internal Parameters and Derivatives of Quasi-Frenergy	217
	References	219
	Subject Index	221