## Contents

| Chapter I   |     | Preliminaries on Categories,             |    |
|-------------|-----|--|----|
| -           |     | Abelian Groups, and Homotopy             | 1  |
|             | § 1 | Categories and Functors                  | 1  |
|             | § 2 | Abelian Groups (Exactness, Direct Sums,  |    |
|             |     | Free Abelian Groups)                     | 7  |
|             | § 3 | Homotopy                                 | 13 |
| Chapter II  |     | Homology of Complexes                    | 16 |
|             | § 1 | Complexes                                | 16 |
|             | § 2 | Connecting Homomorphism,                 |    |
|             |     | Exact Homology Sequence                  | 19 |
|             | § 3 | Chain-Homotopy                           | 23 |
|             | § 4 | Free Complexes                           | 26 |
| Chapter III |     | Singular Homology                        | 29 |
|             | § 1 | Standard Simplices and Their Linear Maps | 29 |
|             | § 2 | The Singular Complex                     | 30 |
|             | § 3 | Singular Homology                        | 32 |
|             | § 4 | Special Cases                            | 33 |
|             | § 5 | Invariance under Homotopy                | 37 |
|             | § 6 | Barycentric Subdivision                  | 40 |
|             | § 7 | Small Simplices. Excision                | 43 |
|             | § 8 | Mayer-Vietoris Sequences                 | 47 |
| Chapter IV  |     | Applications to Euclidean Space          | 54 |
| -           | § 1 | Standard Maps between Cells and Spheres  | 54 |
|             | § 2 | Homology of Cells and Spheres            | 55 |
|             | § 3 | Local Homology                           | 59 |
|             | § 4 | The Degree of a Map                      | 62 |
|             | § 5 | Local Degrees                            | 66 |
|             | § 6 | Homology Properties                      |    |
|             |     | of Neighborhood Retracts in R"           | 71 |



X Contents

|             | § 7   | Jordan Theorem, Invariance of Domain    | 78  |
|-------------|-------|---|-----|
|             | _     | Euclidean Neighborhood Retracts (ENRs)  | 79  |
|             |       |   |     |
| Chapter V   |       | Cellular Decomposition                  |     |
|             |       | and Cellular Homology                   | 85  |
|             |       | Cellular Spaces                         | 85  |
|             |       | CW-Spaces                               | 88  |
|             |       | Examples                                | 95  |
|             | § 4   | Homology Properties of CW-Spaces        | 101 |
|             | -     | The Euler-Poincaré Characteristic       | 104 |
|             | § 6   | Description of Cellular Chain Maps and  |     |
|             |       | of the Cellular Boundary Homomorphism   | 106 |
|             | § 7   | Simplicial Spaces                       | 111 |
|             | § 8   | Simplicial Homology                     | 119 |
| Chapter VI  |       | Functors of Complexes                   | 123 |
|             | § 1   | Modules                                 | 123 |
|             | -     | Additive Functors                       | 127 |
|             |       | Derived Functors                        | 132 |
|             |       | Universal Coefficient Formula           | 136 |
|             | -     | Tensor and Torsion Products             | 140 |
|             |       | Hom and Ext                             | 146 |
|             | -     | Singular Homology and Cohomology        |     |
|             | 0     | with General Coefficient Groups         | 150 |
|             | 8 8   | Tensorproduct and Bilinearity           | 157 |
|             |       | Tensorproduct of Complexes.             |     |
|             | 0 -   | Künneth Formula                         | 161 |
|             | 8 10  | Hom of Complexes.                       | 101 |
|             | 3 - 0 | Homotopy Classification of Chain Maps   | 167 |
|             | 8 1 1 | Acyclic Models                          | 174 |
|             |       | The Eilenberg-Zilber Theorem.           | 17. |
|             | 3 12  | Künneth Formulas for Spaces             | 178 |
| Chantar VII |       | Products                                | 186 |
| Chapter VII | £ 1   | The Scalar Product                      |     |
|             | -     |   | 187 |
|             |       | The Exterior Homology Product           | 189 |
|             | 83    | The Interior Homology Product           | 102 |
|             | 0.4   | (Pontrjagin Product)                    | 193 |
|             |       | Intersection Numbers in IR <sup>n</sup> | 197 |
|             | •     | The Fixed Point Index                   | 202 |
|             | § 6   | The Lefschetz-Hopf Fixed Point          | 00- |
|             |       | Theorem                                 | 207 |
|             | § 7   | The Exterior Cohomology Product         | 214 |

Contents XI

|              | § 8  | The Interior Cohomology Product      |     |
|--------------|------|--------------------------------------|-----|
|              |      | (Product)                            | 219 |
|              | § 9  | Products in Projective Spaces.       |     |
|              |      | Hopf Maps and Hopf Invariant         | 222 |
|              | -    | Hopf Algebras                        | 227 |
|              |      | The Cohomology Slant Product         | 233 |
|              |      | The Cap-Product (~-Product)          | 238 |
|              | § 13 | The Homology Slant Product,          |     |
|              |      | and the Pontrjagin Slant Product     | 245 |
| Chapter VIII |      | Manifolds                            | 247 |
|              | § 1  | Elementary Properties of Manifolds   | 247 |
|              | § 2  | The Orientation Bundle of a Manifold | 251 |
|              | § 3  | Homology of Dimensions $\geq n$      |     |
|              |      | in n-Manifolds                       | 259 |
|              | § 4  | Fundamental Class and Degree         | 266 |
|              | § 5  | Limits                               | 272 |
|              | § 6  | Čech Cohomology                      |     |
|              |      | of Locally Compact Subsets of IR"    | 281 |
|              | § 7  | Poincaré-Lefschetz Duality           | 291 |
|              | § 8  | Examples, Applications               | 298 |
|              | § 9  | Duality in ∂-Manifolds               | 303 |
|              | § 10 | Transfer                             | 308 |
|              | § 11 | Thom Class, Thom Isomorphism         | 314 |
|              |      | The Gysin Sequence. Examples         | 325 |
|              | -    | Intersection of Homology Classes     | 335 |
| Appendix     |      | Kan- and Čech-Extensions of Functors | 348 |
|              | § 1  | Limits of Functors                   | 348 |
|              | § 2  | Polyhedrons under a Space,           |     |
|              |      | and Partitions of Unity              | 352 |
|              | § 3  | Extending Functors from Polyhedrons  |     |
|              | -    | to More General Spaces               | 361 |
|              |      | Bibliography                         | 368 |
|              |      | Subject Index                        | 371 |