

Contents

I Fractal Image Encoding

1	Why Fractal Block Coders Work	3
	<i>Geoffrey Davis</i>	
1.1	Introduction	3
1.2	Linear Transform Coders	4
1.2.1	Entropy Coding	4
1.2.2	The Karhunen-Loève Transform	5
1.2.3	Image Model for Transform Coders	6
1.3	Fractal Block Coders	6
1.3.1	Image Model for Fractal Coders	7
1.3.2	Mechanics of Fractal Block Coders	8
1.4	A Wavelet Framework	9
1.4.1	The Discrete Wavelet Transform	9
1.4.2	A Wavelet Analog of Fractal Block Coding	10
1.4.3	Convergence Criterion	13
1.4.4	Why Fractal Block Coders Work	13
1.5	Self-Quantization of Subtrees	14
1.5.1	Extrapolation in Scale	14
1.5.2	Convergence of Fractal Schemes	16
1.5.3	Results	17
1.6	Conclusion	17
2	On Fractal Compression and Vector Quantization	21
	<i>Skjalg Lepsøy, Paolo Carlini, Geir Egil Øien</i>	
2.1	Two Coders to Compare	21
2.1.1	Product Code Vector Quantization	21
2.1.2	A Mean-Removed Gain-Shape VQ	22
2.1.3	A Fractal Coder	23
2.2	Methods of Codebook Population	24
2.2.1	Population of the Codebook in the Fractal Coder	24
2.2.2	Population of the Codebook in the M-VQ	25
2.3	Cross-Transformability	27
2.3.1	Details on the Experiment	28
2.3.2	Results and Remarks	29

2.4	Trained Codebooks versus Self-Transformation	29
2.4.1	Details on the Experiment	29
2.4.2	Results and Remarks	35
2.4.3	Characteristics of the Two Techniques	36
2.4.4	Conclusion	44
2.5	Discussion	44
2.5.1	Other Segmentation Schemes	46
2.5.2	Position-Dependent Codebooks	46
2.6	Conclusion	47
3	On the Use of Subsampling in Fractal Image Compression	49
	<i>Lars Lundheim</i>	
3.1	Introduction	49
3.2	Notation and Terminology	50
3.3	Decimation by Subsampling	51
3.4	The Family Graph	52
3.5	An Efficient Synthesis Algorithm	55
3.6	Complexity	56
3.7	An Alternative Algorithm	58
3.8	Empirical Results	59
3.9	Contractivity	61
3.9.1	Test of Eventual Contractivity	63
3.10	Conclusion	64
4	On the Benefits of Basis Orthogonalization in Fractal Compression	65
	<i>Geir E. Øien and Skjalg Lepsøy</i>	
4.1	Introduction	65
4.2	Introducing Basis Orthogonalization in the Fractal Signal Model	66
4.3	Decoder Convergence	69
4.3.1	Convergence Speed	69
4.3.2	Invariance of Attractor	71
4.3.3	Similarity Between Collage and Attractor	72
4.3.4	Pyramid Decoder Structure	75
4.4	Fractal Model Coefficients	78
4.4.1	Coefficient Optimization	78
4.4.2	Coefficient and Offset Subspace Properties	79
4.4.3	Image Coding: Practical Considerations and Compromises	81
4.5	Encoder Complexity Reduction	82
4.5.1	Simplified Coefficient Computation	82
4.5.2	Simplified Distortion Minimization	82
4.5.3	Directed Domain Pool Search	82
4.6	Comparisons to Other Techniques	85
4.6.1	Product Code Vector Quantization	85
4.6.2	Other Lossy Compression Techniques	86
4.7	Conclusion	87

5	On the Dimension of Fractally Encoded Images	89
	<i>Yuval Fisher, Frank Dudbridge, Ben Bielefeld</i>	
5.1	Introduction	89
5.2	The Fractal Dimension of Image Blocks	90
5.3	Results	92
5.4	Conclusion	94
5.5	Acknowledgments	94
6	Fractal Image Compression via Nearest Neighbor Search	95
	<i>Dietmar Saupe</i>	
6.1	Introduction	96
6.1.1	The Search Problem in Fractal Image Encoding	96
6.1.2	Previous Work	97
6.2	A Formula for the Least Squares Error Based on Projections	98
6.3	Searching in Fractal Image Compression is Nearest Neighbor Search	100
6.4	Practical Considerations	103
6.5	Implementation	104
6.6	Results	106
6.6.1	Choosing Parameters for the Approximate Nearest Neighbor Search	106
6.6.2	Fast Nearest Neighbor Search with Classification	106
6.6.3	Fast Nearest Neighbor Search without Loss in Fidelity	108
6.6.4	On the Use of Orthonormally Transformed Feature Space	109
6.6.5	Quantization Issues	110
6.7	Other Work Related to Feature Vectors	112
6.7.1	Straight Feature Vectors	113
6.7.2	Invariant Moments	113
6.7.3	Tree Structured Methods	114
6.8	Conclusion	114
7	Fractal Image Coding: Some Mathematical Remarks on Its Limits and Its Prospects	117
	<i>F.M. Dekking</i>	
7.1	Introduction	117
7.2	Basics	118
7.3	Coding of 1-D Black and White Images	119
7.4	Grey Value Coding: The Collage and the Attractor	122
7.5	Pyramids and Martingales in Grey Value Image Coding	126
7.6	Martingales for Electrical Engineers	128
8	Linear Time Fractal Quadtree Coder	133
	<i>F. Dudbridge</i>	
8.1	Introduction	133
8.2	A Model for Fractal Images	133
8.3	Image Coding	134
8.4	Reconstruction	135
8.5	Implementation	135

9	Fractal Encoding of Video Sequences	139
	<i>Yuval Fisher</i>	
9.1	Introduction and Notation	139
9.2	Video Sequences	140
9.3	Implementation	141
9.4	Results	142
9.5	Discussion	143
9.6	Conclusion	143
10	Theory of Generalized Fractal Transforms	145
	<i>Bruno Forte and Edward R. Vrscay</i>	
10.1	Introduction	145
10.2	Generalized Fractal Transforms	147
10.3	From IFS to IFSM Fractal Transforms	150
10.3.1	IFS	150
10.3.2	IFZS	152
10.3.3	IFSM	154
10.3.4	IFSP	156
10.4	IFS on the Space of Distributions $\mathcal{D}'(X)$	159
10.4.1	Affine IFSD and the Connection with IFSP and IFSM	164
10.4.2	Integrals Involving Affine IFSD	166
11	Inverse Problem Methods for Generalized Fractal Transforms	169
	<i>Bruno Forte and Edward R. Vrscay</i>	
11.1	Introduction	169
11.2	Inverse Problem for IFSM	170
11.2.1	Collage Theorem for IFSM in $\mathcal{L}^2(X, \mu)$	170
11.2.2	Formal Solution to the IFSM Inverse Problem	173
11.2.3	"Local IFSM" on $\mathcal{L}^p(X, m)$	176
11.2.4	Inverse Problem With Place-Dependent IFSM	177
11.2.5	Application of IFSM Methods to Images	179
11.3	Approximation of Measures Using IFSP	180
11.3.1	Approximation by "Moment Matching"	182
11.3.2	Collage Theorem for Moments	183
11.3.3	Formal Solution to Inverse Problem	186
11.3.4	Some Numerical Results	187
11.4	IFSM and Fractal Wavelet Compression	189
11.5	Collage Theorem for Fourier Transforms	196
12	Fractal Compression of ECG Signals	201
	<i>Geir E. Øien and Geir Nørstad</i>	
12.1	Introduction to ECG Signals	201
12.1.1	ECG Measurement and Characteristics	202
12.1.2	Digitization and Storage of ECG Signals	203
12.1.3	History of ECG Compression – a Brief Review	205
12.2	Fractal Compression and ECG Signals	205

12.2.1	Redundancy and Irrelevance	205
12.2.2	Why Should Fractal Compression Work?	206
12.2.3	The Fractal Signal Model	206
12.2.4	Parameter Choices	208
12.2.5	Parameter Optimization	210
12.2.6	The Coefficient Quantization Scheme	211
12.2.7	The Decoding Scheme	216
12.3	Results and Discussion	216
12.3.1	Quality Criteria	216
12.3.2	Bit Allocation Experiments	217
12.3.3	Quantizer Characteristics	217
12.3.4	Coding Experiments	217
12.3.5	Comparisons to Other Techniques	225
12.4	Conclusion	225

II Fractal Image Analysis

13 Dimensions of Fractals and Multifractals 229

Kenneth Falconer

13.1	Dimensions of Fractals	229
13.1.1	Hausdorff and Packing Dimensions	229
13.1.2	Calculation of Dimensions	231
13.1.3	Iterated Function Systems	232
13.1.4	Self-similar Sets	233
13.1.5	Self-affine Sets	234
13.1.6	Related Constructions	235
13.2	Fractal Measures	236
13.2.1	Local Dimensions and Multifractal Spectra	236
13.2.2	Invariant Measures	237
13.2.3	The Multifractal Spectrum of Self-similar Measures	237
13.2.4	Other Examples	239
13.2.5	Other Approaches to Multifractal Theory	240

14 Velocity, Length, Dimension 243

Claude Tricot

14.1	Introduction	243
14.2	Finite Length	244
14.3	Infinite Length: A Geometrical Approach	246
14.4	The Metric Set of Intervals: Preliminary Results	247
14.5	A Generalized Notion of Length	248
14.6	Infinite Length: An Analytical Approach	250
14.7	The Compass Method	251
14.8	The Method of Constant Breadth	252
14.9	Comparison of Lengths	253
14.10	Fractal Dimension of Curves	254

14.11 A Few Typical Curves	255
14.11.1 Rectifiable curves	255
14.11.2 Spirals	255
14.11.3 Self-similarity	257
14.11.4 Self-affine graphs	257
14.11.5 Graphs of continuous functions	258
14.11.6 Other curve with uniform breadth	258
15 A Local Multiscale Characterization of Edges	
Applying the Wavelet Transform	261
<i>Carl J.G. Evertsz, Kathrin Berkner, Wilhelm Berghorn</i>	
15.1 Introduction	261
15.2 Gaussian Kernel Smoothing	262
15.3 Edge Detection and Filtering with Wavelets	264
15.3.1 The Wavelet Transform	264
15.3.2 Hölder Exponents of Singularities and Edges	265
15.3.3 Edge-Parameters	267
15.4 Numerical Implementation	267
15.4.1 Finding Modulus-Maxima Lines	267
15.4.2 Estimating Hölder Exponents and Other Edge-Parameters	270
15.4.3 Noise	271
15.5 Applications	272
15.5.1 Bone Tumors	272
15.5.2 Liver Tumor	272
15.5.3 Lena	272
15.5.4 Conclusions and Summary	275
16 Local Connected Fractal Dimension Analysis of Early Chinese Landscape Paintings and X-Ray Mammograms	279
<i>Richard F. Voss</i>	
16.1 Fractals: From Self-Similarity to Self-Affinity and Multifractals	279
16.2 Measures and Mass Dimension	280
16.3 Local Image Gradient and the Statistics of Natural Images	283
16.4 Local Fractal Mass Dimension	283
16.5 Classification of Early Chinese Landscape Drawings	286
16.6 Local Connected Fractal Dimension	286
16.7 Preliminary Classification of X-ray Mammograms	290
16.8 Discussion	295
17 Introduction to the Multifractal Analysis of Images	299
<i>Jacques Lévy-Véhel</i>	
17.1 Introduction	299
17.2 A Quick Review on Some “Classical” Methods	300
17.2.1 Canny’s Edge Detector	300
17.2.2 Mathematical Morphology	301
17.2.3 Image Multiscale Analysis	305

17.3	The Multifractal Approach	309
17.3.1	Overview	309
17.3.2	Choquet Capacities	310
17.3.3	Multifractal Analysis of Sequences of Choquet Capacities . . .	311
17.3.4	An Example: the Binomial Measure	314
17.3.5	Multifractal Correlations	319
17.4	Application to Image Analysis	321
17.4.1	Introduction	321
17.4.2	Numerical Estimation of f_g	325
17.4.3	Image Enhancement	329
17.4.4	Change Detection	331
17.4.5	Edge Detection	331
Bibliography		343
List of Participants		355
Index		361