

CONTENTS

Acknowledgements	i
Zusammenfassung	iii
Abstract	vii
List of Figures	xi
1 Introduction	1
2 Notation and foundations	9
3 From cutting planes to lattice-free polyhedra	13
3.1 Properties of $\text{conv}(P_I)$	14
3.2 Facets of $\text{conv}(P_I)$ and lattice-free polyhedra	19
4 Evaluation of cutting planes	23
4.1 A negative result on the strength	26
4.2 A positive result on the strength	29
5 Area – lattice width relations in the plane	35
5.1 Preliminaries	36
5.2 Main results	41
5.3 Auxiliary results on triangles	46

5.4	Proofs for arbitrary sets	49
5.5	Proofs for centrally symmetric sets	61
6	A probabilistic model for the evaluation of cutting planes	69
6.1	Motivation	70
6.2	Probabilistic model and main results	72
6.3	Type 1 triangles	75
6.4	Strategy for triangles of types 2 and 3, and quadrilaterals . .	76
6.5	Type 2 triangles	78
6.5.1	Regions R_3 and R_4	80
6.5.2	Regions R_5 and R_6	80
6.5.3	Regions R_1 and R_2	80
6.5.4	Approximation for $P^{T_2}(z)$	81
6.6	Quadrilaterals	85
6.6.1	Regions R_1 and R_2	87
6.6.2	Regions R_3 and R_4	88
6.6.3	Approximation for $P^Q(z)$	89
6.7	Type 3 triangles	90
7	On finiteness of lattice-free polyhedra	95
7.1	Preliminaries and main results	97
7.2	Proof of Theorem 7.2	99
7.3	Remarks on the volume bound	107
7.4	The relation between $\mathcal{P}_{\text{fmi}}^d(s)$ and $\mathcal{P}_{\text{ifm}}^d(s)$	109
8	Three-dimensional maximal lattice-free integral polyhedra	115
8.1	Preliminaries and proof outline	117
8.2	Elements in \mathcal{M}^3 with six facets	120
8.3	Elements in \mathcal{M}^3 with five facets	123
8.3.1	Quadrangular pyramids	123
8.3.2	Triangular prisms	131
8.4	Elements in \mathcal{M}^3 with four facets	133
8.5	Remarks on the computer enumeration	140
	Outlook	143
	Bibliography	147
	Index	155