

CONTENTS

PART I: QUANTUM GRAVITY	1
1. QUANTUM GRAVITY, QUANTUM COSMOLOGY AND CLASSICAL GRAVITY	3
1.1 Quantum Gravity: Approaches, Achievements and Unsolved Problems	3
1.2 Quantum Cosmology: Motivations and Some Recent Developments	9
1.3 Introduction to Supergravity	13
1.4 An Outline of This Work	16
2. CANONICAL QUANTUM GRAVITY	27
2.1 Hamiltonian Methods in Physics	28
2.2 Dirac's Quantization of First-Class Constrained Hamiltonian Systems	35
2.3 Dirac's Quantization of Second-Class Constrained Hamiltonian Systems	37
2.4 ADM Formalism and Constraints in Canonical Quantum Gravity	43
2.5 Mathematical Theory of Wheeler's Superspace	49
3. PERTURBATIVE QUANTUM GRAVITY	53
3.1 The One-Loop Approximation	55
3.2 Zeta-Function Regularization of Path Integrals	59
3.3 Gravitational Instantons	61
3.4 Perturbative Renormalization of Quantum Field Theories	71

PART II: ONE-LOOP QUANTUM COSMOLOGY	77
 4. GLOBAL BOUNDARY CONDITIONS AND $\zeta(0)$ VALUE	
FOR THE MASSLESS SPIN-$\frac{1}{2}$ FIELD	79
4.1 Physical-Degrees-of-Freedom One-Loop Results for Pure Gravity	80
4.2 Mathematical Foundations of Global Boundary Conditions	84
4.3 How to Deal with First-Order Differential Operators	89
4.4 Detailed Calculation of the Infinite Sums	92
4.5 The Heat Kernel and the Prefactor	98
4.6 From Global to Local Boundary Conditions	100
 5. CHOICE OF BOUNDARY CONDITIONS IN ONE-LOOP	
QUANTUM COSMOLOGY	103
5.1 General Form of the Action of the Spin- $\frac{3}{2}$ Field	105
5.2 Hamiltonian Form of the Action and Supersymmetry Constraints	107
5.3 Gauge Condition	109
5.4 Final Form of the Action	112
5.5 PDF Prefactor of the Semiclassical Wave Function	
with Global Boundary Conditions	112
5.6 Basic Results of Twistor Theory in Flat Space	117
5.7 Local Boundary Conditions and Spin-Lowering Operators	119
5.8 Application of Local Boundary Conditions to the Spin- $\frac{1}{2}$ Field	124
5.9 Spin-1 and Spin-0 Fields	138
5.10 Preservation in Time of the Gauge Constraint	150
 6. GHOST FIELDS AND GAUGE MODES IN ONE-LOOP	
QUANTUM COSMOLOGY	152
6.1 Main Ideas About Gauge Theories and Their Quantization	154
6.2 Extended Phase Space for the Spin-1 Field	156
6.3 Formal Equivalence to the Faddeev-Popov Result	159
6.4 Physical Degrees of Freedom and Redundant Variables	160
6.5 A More Careful Study of the Spin-1 Problem	164
 ADDENDUM TO CHAPTER SIX	174

7. LOCAL BOUNDARY CONDITIONS FOR THE WEYL SPINOR	188
7.1 Local Boundary Conditions for the Spin-2 Field Strength	190
7.2 One Cannot Fix the Linearized Electric Curvature on S^3	191
7.3 Calculation of the PDF $\zeta(0)$ when the Linearized Magnetic Curvature is Vanishing on S^3	200
7.4 Comparison of Different Techniques	210
8. ONE-LOOP RESULTS FOR THE SPIN-$\frac{1}{2}$ FIELD WITH LOCAL BOUNDARY CONDITIONS	214
8.1 General Structure of the $\zeta(0)$ Calculation for the Spin- $\frac{1}{2}$ Field Subject to Local Boundary Conditions on S^3	215
8.2 Contribution of $W_\infty^{(1)}$ and $W_\infty^{(2)}$	221
8.3 Effect of $W_\infty^{(3)}, W_\infty^{(4)}$ and $W_\infty^{(5)}$	222
8.4 Vanishing Effect of Higher-Order Terms	223
8.5 $\zeta(0)$ Value	225
9. LOCAL SUPERSYMMETRY IN PERTURBATIVE QUANTUM COSMOLOGY	230
9.1 Local Boundary Conditions for the Spin- $\frac{3}{2}$ Potential	231
9.2 Physical-Degrees-of-Freedom Contribution to $\zeta(0)$	234
9.3 Problems for Extended Supergravity Theories	238
PART III: CLASSICAL GRAVITY	243
10. LORENTZIAN GEOMETRY, U_4 THEORIES AND SINGULARITIES IN COSMOLOGY	245
10.1 Introduction	247
10.2 Lorentzian and Riemannian Geometry	248
10.3 Spinor Structure	252
10.4 Causal Structure	255
10.5 Conformal Structure and Asymptotic Properties	262
10.6 Hamiltonian Structure	278
10.7 The Singularity Problem for Space-Times with Torsion	282
10.8 Concluding Remarks	294

Contents

PART IV: SUMMARY	301
11. CONCLUSIONS	303
11.1 Foundational Issues	303
11.2 Our Results	305
11.3 Our Unsolved Problems	307
 PROBLEMS FOR THE READER	 310
 APPENDIX A: Two-Component Spinor Calculus and Its Applications	 314
APPENDIX B: The Generalized Zeta-Function	321
APPENDIX C: Euler-Maclaurin Formula and Free Part of the Heat Kernel for the Spin- $\frac{3}{2}$ Field	323
APPENDIX D: Complex Manifolds	326
APPENDIX E: Lorentzian ADM Formulae for the Curvature	328
APPENDIX F: $\zeta(0)$ Calculations	330
 REFERENCES	 335