

Contents

Part I Basic Notions and Review

1	Dynamical Systems and Hyperbolicity	3
1.1	Dynamical systems: basic notions	3
1.1.1	Systems with continuous and discrete time, and their mutual relation	3
1.1.2	Dynamics in terms of phase fluid: Conservative and dissipative systems and attractors	6
1.1.3	Rough systems and structural stability	8
1.1.4	Lyapunov exponents and their computation	10
1.2	Model examples of chaotic attractors	12
1.2.1	Chaos in terms of phase fluid and baker's map	12
1.2.2	Smale-Williams solenoid	15
1.2.3	DA-attractor	16
1.2.4	Plykin type attractors	17
1.3	Notion of hyperbolicity	19
1.4	Content and conclusions of the hyperbolic theory	22
1.4.1	Cone criterion	24
1.4.2	Instability	25
1.4.3	Transversal Cantor structure and Kaplan-Yorke dimension	25
1.4.4	Markov partition and symbolic dynamics	26
1.4.5	Enumerating of orbits and topological entropy	27
1.4.6	Structural stability	28
1.4.7	Invariant measure of Sinai-Ruelle-Bowen	29
1.4.8	Shadowing and effect of noise	30
1.4.9	Ergodicity and mixing	30
1.4.10	Kolmogorov-Sinai entropy	31
	References	31

2	Possible Occurrence of Hyperbolic Attractors	35
2.1	The Newhouse-Ruelle-Takens theorem and its relation to the uniformly hyperbolic attractors	35
2.2	Lorenz model and its modifications	37
2.3	Some maps with uniformly hyperbolic attractors	40
2.4	From DA to the Plykin type attractor	43
2.5	Hunt's example: Suspending the Plykin type attractor	46
2.6	The triple linkage: A mechanical system with hyperbolic dynamics	49
2.7	A possible occurrence of a Plykin type attractor in Hindmarsh-Rose neuron model	51
2.8	Blue sky catastrophe and birth of the Smale-Williams attractor	52
2.9	Taffy-pulling machine	53
	References	54

Part II Low-Dimensional Models

3	Kicked Mechanical Models and Differential Equations with Periodic Switch	59
3.1	Smale-Williams solenoid in mechanical model: Motion of a particle on a plane under periodic kicks	60
3.2	A set of switching differential equations with attractor of Smale-Williams type	65
3.3	Explicit dynamical system with attractor of Plykin type	68
3.3.1	Plykin type attractor on a sphere	68
3.3.2	Plykin type attractor on the plane	73
3.4	Plykin-like attractor in smooth non-autonomous system	76
	References	79
4	Non-Autonomous Systems of Coupled Self-Oscillators	81
4.1	Van der Pol oscillator	81
4.2	Smale-Williams attractor in a non-autonomous system of alternately excited van der Pol oscillators	84
4.3	System of alternately excited van der Pol oscillators in terms of slow complex amplitudes	93
4.4	Non-resonance excitation transfer	94
4.5	Plykin-like attractor in non-autonomous coupled oscillators	95
4.5.1	Representation of states on a sphere and equations of the model	95
4.5.2	Numerical results for the coupled oscillators	98
	References	101
5	Autonomous Low-dimensional Systems with Uniformly Hyperbolic Attractors in the Poincaré Maps	103
5.1	Autonomous system of two coupled oscillators with self-regulating alternating excitation	103

5.2	System constructed on a base of the predator-prey model	107
5.3	Example of blue sky catastrophe accompanied by a birth of Smale-Williams attractor	112
	References	117
6	Parametric Generators of Hyperbolic Chaos	119
6.1	Parametric excitation of coupled oscillators. Three-frequency parametric generator and its operation	120
6.2	Hyperbolic chaos in parametric oscillator with Q-switch and pump modulation	123
6.2.1	Dynamical equations	123
6.2.2	Qualitative explanation of the operation	126
6.2.3	Numerical results	127
6.2.4	Numerical results in the frame of method of slow complex amplitudes	129
6.3	Parametric generator of hyperbolic chaos based on four coupled oscillators with pump modulation	131
6.3.1	Model, operation principle and basic equations	132
6.3.2	Chaotic dynamics: results of computer simulation	134
	References	139
7	Recognizing the Hyperbolicity: Cone Criterion and Other Approaches	141
7.1	Verification of transversality for manifolds	141
7.1.1	Visualization of the manifolds	142
7.1.2	Distributions of angles of the manifold intersections	144
7.2	Visualization of invariant measures	150
7.3	Cone criterion and examples of its application	155
7.3.1	Procedure of verification of the cone criterion	155
7.3.2	Examples of application of the cone criterion	161
	References	169
Part III Higher-Dimensional Systems and Phenomena		
8	Systems of Four Alternately Excited Non-autonomous Oscillators	173
8.1	Arnold's cat map dynamics in a system of coupled non-autonomous van der Pol oscillators	173
8.2	Dynamics corresponding to hyperchaotic maps	180
8.2.1	System implementing toral hyperchaotic map	180
8.2.2	Model with cascade transfer of excitation upward the frequency spectrum	182
8.3	Hyperchaos and synchronous chaos in a system of coupled non-autonomous oscillators	187
8.3.1	Equations and basic modes of operation	188
8.3.2	Equations for slow complex amplitudes	193
	References	199

9	Autonomous Systems Based on Dynamics Close to Heteroclinic Cycle	201
9.1	Heteroclinic connection: an example of Guckenheimer and Holmes	201
9.2	Attractor of Smale-Williams type in a system of three coupled self-oscillators	203
9.3	Attractor with dynamics governed by the Arnold cat map	207
9.4	Model with hyperchaos	210
9.5	An autonomous system with attractor of Smale-Williams type with resonance transfer of excitation in a ring array of van der Pol oscillators	213
	References	217
10	Systems with Time-delay Feedback	219
10.1	Some notions concerning differential equations with deviating argument	220
10.2	Van der Pol oscillator with delayed feedback, parameter modulation and auxiliary signal	223
10.2.1	Attractor of Smale-Williams type in the time-delayed system	224
10.2.2	Hyperchaotic attractors	227
10.3	Van der Pol oscillator with two delayed feedback loops and parameter modulation	231
10.4	Autonomous time-delay system	237
	References	240
11	Chaos in Co-operative Dynamics of Alternately Synchronized Ensembles of Globally Coupled Self-oscillators	243
11.1	Kuramoto transition in ensemble of globally coupled oscillators	243
11.2	Model of two alternately synchronized ensembles of oscillators	247
11.2.1	Collective chaos in ensemble of van der Pol oscillators	248
11.2.2	Slow-amplitude approach	251
11.2.3	Description of the dynamics in terms of ensembles of phase oscillators	254
	References	256
Part IV Experimental Studies		
12	Electronic Device with Attractor of Smale-Williams Type	259
12.1	Scheme of the device and the principle of operation	259
12.2	Experimental observation of the Smale-Williams attractor	260
	References	263

13 Delay-time Electronic Devices Generating Trains of Oscillations with Phases Governed by Chaotic Maps	265
13.1 Van der Pol oscillator with delayed feedback, parameter modulation and auxiliary signal	265
13.2 Van der Pol oscillator with two delayed feedback loops and parameter modulation	269
References	272
14 Conclusion	273
References	275
Appendix A Computation of Lyapunov Exponents: The Benettin Algorithm	277
References	279
Appendix B Hénon and Ikeda Maps	281
References	287
Appendix C Smale's Horseshoe and Homoclinic Tangle	289
References	292
Appendix D Fractal Dimensions and Kaplan-Yorke Formula	293
References	297
Appendix E Hunt's Model: Formal Definition	299
References	303
Appendix F Geodesics on a Compact Surface of Negative Curvature	305
References	309
Appendix G Effect of Noise in a System with a Hyperbolic Attractor	311
References	317
Index	319