

Contents

Part I Foundations of the Theory of Waves in Nondispersive Media	1
1 Nonlinear Equations of the First Order	3
1.1 Simple wave equation	3
1.1.1 The canonical form of the equation	3
1.1.2 Particle flow	4
1.1.3 Discussion of the Riemann solution	5
1.1.4 Compressions and expansions of the particle flow	6
1.1.5 Continuity equation	8
1.1.6 Construction of the density field	9
1.1.7 Momentum-conservation law	10
1.1.8 Fourier transforms of density and velocity	11
1.2 Line-growth equation	13
1.2.1 Forest-fire propagation	13
1.2.2 Anisotropic surface growth	16
1.2.3 Solution of the surface-growth equation	18
1.3 One-dimensional laws of gravitation	20
1.3.1 Lagrangian description of one-dimensional gravitation	20
1.3.2 Eulerian description of one-dimensional gravitation	22
1.3.3 Collapse of a one-dimensional Universe	24
1.4 Problems to Chapter 1	25
References	36
2 Generalized Solutions of Nonlinear Equations	39
2.1 Standard equations	39
2.1.1 Particle-flow equations	40
2.1.2 Line growth in the small angle approximation	40
2.1.3 Nonlinear acoustics equation	41
2.2 Multistream solutions	42
2.2.1 Interval of single-stream motion	42

2.2.2	Appearance of multistreamness	42
2.2.3	Gradient catastrophe	44
2.3	Sum of streams	46
2.3.1	Total particle flow	46
2.3.2	Summation of streams by inverse Fourier transform	47
2.3.3	Algebraic sum of the velocity field	47
2.3.4	Density of a “warm” particle flow	48
2.4	Weak solutions of nonlinear equations of the first order	50
2.4.1	Forest fire	50
2.4.2	The Lax-Oleinik absolute minimum principle	52
2.4.3	Geometric construction of weak solutions	53
2.4.4	Convex hull	54
2.4.5	Maxwell’s rule	56
2.5	The E-Rykov-Sinai global principle	59
2.5.1	Flow of inelastically coalescing particles	59
2.5.2	Inelastic collisions of particles	60
2.5.3	Formulation of the global principle	61
2.5.4	Mechanical meaning of the global principle	62
2.5.5	Condition of physical realizability	63
2.5.6	Geometry of the global principle	66
2.5.7	Solutions of the continuity equation	69
2.6	Line-growth geometry	70
2.6.1	Parametric equations of a line	71
2.6.2	Contour in polar coordinates	72
2.6.3	Contour envelopes	74
2.7	Problems to Chapter 2	77
	References	82
3	Nonlinear Equations of the Second Order	83
3.1	Regularization of nonlinear equations	83
3.1.1	The Kardar-Parisi-Zhang equation	84
3.1.2	The Burgers equation	85
3.2	Properties of the Burgers equation	86
3.2.1	Galilean invariance	86
3.2.2	Reynolds number	87
3.2.3	Hubble expansion	89
3.2.4	Stationary wave	92
3.2.5	Khokhlov’s solution	93
3.2.6	Rudenko’s solution	95
3.3	General solution of the Burgers equation	100
3.3.1	The Hopf-Cole substitution	101
3.3.2	General solution of the Burgers equation	102
3.3.3	Averaged Lagrangian coordinate	103
3.3.4	Solution of the Burgers equation with vanishing viscosity	104

3.4	Model equations of gas dynamics	105
3.4.1	One-dimensional model of a polytropic gas	105
3.4.2	Discussion of physical properties of a model gas	108
3.5	Problems to Chapter 3	111
	References	115
4	Field Evolution Within the Framework of the Burgers Equation	117
4.1	Evolution of one-dimensional signals	117
4.1.1	Self-similar solution, once more	117
4.1.2	Approach to the linear stage	119
4.1.3	N -wave and U -wave	120
4.1.4	Sawtooth waves	123
4.1.5	Periodic waves	128
4.2	Evolution of complex signals	131
4.2.1	Quasiperiodic complex signals	132
4.2.2	Evolution of fractal signals	133
4.2.3	Evolution of multi-scale signals — a dynamic turbulence model	135
4.3	Problems to Chapter 4	147
	References	152
5	Evolution of a Noise Field Within the Framework of the Burgers Equation	153
5.1	Burgers turbulence — acoustic turbulence	153
5.2	The Burgers turbulence at the initial stage of evolution	155
5.2.1	One-point probability density of a random Eulerian velocity field	157
5.2.2	Properties of the probability density of a random velocity field	159
5.2.3	Spectra of a velocity field	162
5.3	Turbulence evolution at the stage of developed discontinuities	166
5.3.1	Phenomenology of the Burgers turbulence	167
5.3.2	Evolution of the Burgers turbulence: statistically homogeneous potential and velocity ($n > 1$ and $n < -3$) ...	171
5.3.3	Exact self-similarity ($n > 2$)	173
5.3.4	Violation of self-similarity ($1 < n < 2$)	176
5.3.5	Evolution of turbulence: statistically inhomogeneous potential ($-3 < n < 1$)	178
5.3.6	Statistically homogeneous velocity and inhomogeneous potential ($-1 < n < 1$)	179
5.3.7	Statistically inhomogeneous velocity and inhomogeneous potential ($-3 < n < -1$)	181
5.3.8	Evolution of intense acoustic noise	182
	References	185

6	Multidimensional Nonlinear Equations	189
6.1	Nonlinear equations of the first order	189
6.1.1	Main equations of three-dimensional flows	189
6.1.2	Lagrangian and Eulerian description of a three-dimensional flow	191
6.1.3	Jacobian matrix for the transformation from Lagrangian to Eulerian coordinates	192
6.1.4	Density of a multidimensional flow	193
6.1.5	Weak solution of the surface-growth equation	194
6.1.6	Flows of locally interacting particles and a singular density field	197
6.2	Multidimensional nonlinear equations of the second order	201
6.2.1	The two-dimensional KPZ equation	201
6.2.2	The three-dimensional Burgers equation	202
6.2.3	Model density field	203
6.2.4	Concentration field	204
6.3	Evolution of the main perturbation types in the KPZ equation and in the multidimensional Burgers equation	207
6.3.1	Asymptotic solutions of the multidimensional Burgers equation and local self-similarity	208
6.3.2	Evolution of simple localized perturbations	212
6.3.3	Evolution of periodic structures under infinite Reynolds numbers	214
6.3.4	Evolution of the anisotropic Burgers turbulence	219
6.3.5	Evolution of perturbations with complex internal structure	225
6.3.6	Asymptotic long-time behavior of a localized perturbation	231
6.3.7	Appendix to Section 6.3. Statistical properties of maxima of inhomogeneous random Gaussian fields	233
6.4	Model description of evolution of the large-scale structure of the Universe	236
6.4.1	Gravitational instability in an expanding Universe	236
6.4.2	From the Vlasov-Poisson equation to the Zeldovich approximation and adhesion model	238
	References	243

Part II Mathematical Models and Physical Phenomena in Nonlinear Acoustics 245

7	Model Equations and Methods of Finding Their Exact Solutions	247
7.1	Introduction	247
7.1.1	Facts from the linear theory	247
7.1.2	How to add nonlinear terms to simplified equations	253
7.1.3	More general evolution equations	255

7.1.4	Two types of evolution equations	256
7.2	Lie groups and some exact solutions	257
7.2.1	Exact solutions of the Burgers equation	257
7.2.2	Finding exact solutions of the Burgers equation by using the group-theory methods	259
7.2.3	Some methods of finding exact solutions	261
7.3	The <i>a priori</i> symmetry method	266
	References	268
8	Types of Acoustic Nonlinearities and Methods of Nonlinear Acoustic Diagnostics	271
8.1	Introduction	271
8.1.1	Physical and geometric nonlinearities	271
8.2	Classification of types of acoustic nonlinearity	274
8.2.1	Boundary nonlinearities	275
8.3	Some mechanisms of bulk structural nonlinearity	280
8.3.1	Nonlinearity of media with strongly compressible inclusions	281
8.3.2	Nonlinearity of solid structurally inhomogeneous media	284
8.4	Nonlinear diagnostics	290
8.4.1	Inverse problems of nonlinear diagnostics	292
8.4.2	Peculiarities of nonlinear diagnostics problems	294
8.5	Applications of nonlinear diagnostics methods	297
8.5.1	Detection of bubbles in a liquid and cracks in a solid	297
8.5.2	Measurements based on the use of radiation pressure	299
8.5.3	Nonlinear acoustic diagnostics in construction industry	300
8.6	Non-typical nonlinear phenomena in structurally inhomogeneous media	301
	References	304
9	Nonlinear Sawtooth Waves	309
9.1	Sawtooth waves	309
9.2	Field and spectral approaches in the theory of nonlinear waves	312
9.2.1	General remarks	312
9.2.2	Generation of harmonics	313
9.2.3	Degenerate parametric interaction	314
9.3	Diffraction beams of sawtooth waves	318
9.4	Waves in inhomogeneous media and nonlinear geometric acoustics	323
9.5	The focusing of discontinuous waves	328
9.6	Nonlinear absorption and saturation	335
9.7	Kinetics of sawtooth waves	340
9.8	Interaction of waves containing shock fronts	344
	References	350

10	Self-action of Spatially Bounded Waves Containing Shock Fronts . . .	357
10.1	Introduction	357
10.2	Self-action of sawtooth ultrasonic wave beams due to the heating of a medium and acoustic wind formation	359
10.3	Self-refraction of weak shock waves in a quadratically nonlinear medium	367
10.4	Non-inertial self-action in a cubically nonlinear medium	373
10.5	Symmetries and conservation laws for an evolution equation describing beam propagation in a nonlinear medium	379
10.6	Conclusions	384
	References	385
11	Nonlinear Standing Waves, Resonance Phenomena and Frequency Characteristics of Distributed Systems	389
11.1	Introduction	389
11.2	Methods of evaluation of the characteristics of nonlinear resonators	390
11.3	Standing waves and the Q-factor of a resonator filled with a dissipating medium	395
11.4	Frequency responses of a quadratically nonlinear resonator	400
11.5	Q-factor increase under introduction of losses	408
11.6	Geometric nonlinearity due to boundary motion	412
11.7	Resonator filled with a cubically nonlinear medium	423
	References	437
	Appendix Fundamental Properties of Generalized Functions	441
A.1	Definition of generalized functions	441
A.2	Fundamental sequences	443
A.3	Derivatives of generalized functions	449
A.4	The Leibniz formula	450
A.5	Derivatives of discontinuous functions	453
A.6	Generalized functions of a composite argument	456
A.7	Multidimensional generalized functions	458
A.8	Continuity equation	463
	A.8.1 Singular solution	463
	A.8.2 Green's function	464
	A.8.3 Lagrangian and Eulerian coordinates	466
A.9	Method of characteristics	466
	Index	471