

Contents

1. Introduction	1
2. Formulation of Elastodynamic Problems.	
Some General Results	4
2.1 Fundamental Equations of Elastodynamics	4
2.2 Initial and Boundary Conditions. Interfaces	6
2.3 Constraints Imposed on the Solution Behavior in the Neighborhood of Singular Points/Curves	9
2.4 Continuous and Discontinuous Solutions	12
2.5 Uniqueness Theorem for Solutions to Elastodynamic Problems with Strong Discontinuities	15
2.6 The Green–Volterra Formula	21
2.7 Various Representations of Solutions to the Equations of Motion of a Homogeneous Isotropic Medium	26
2.7.1 Lamé Representation	26
2.7.2 The Case of a Separable Solution to the Vector Wave Equation	31
2.7.3 Iacovache’s Representation	37
2.7.4 Representation Employing Papkovitch–Neuber Functions	39
2.8 On the Relationships Between Solutions of Transient Dynamic Problems and Those of Static, Steady-State and Stationary Dynamic Problems	40
3. The Method of Functionally Invariant Solutions (the Smirnov–Sobolev Method)	43
3.1 Functionally Invariant Solutions to the Wave Equation	44
3.2 Plane and Complex Waves	46
3.2.1 Reflection of Plane Longitudinal and Transverse Waves	47
3.2.2 The Case of Total Internal Reflection	52
3.2.3 Rayleigh Waves	55
3.3 Homogeneous Solutions	57
3.3.1 Construction of Homogeneous Solutions to the Wave Equation	57
3.3.2 Diffraction of a Plane Shear Wave by a Wedge	61

3.4	The Case of an Elastic Half-Plane	64
3.4.1	Reduction of the Dynamic Problem to Superposition of Problems for Symmetric and Antisymmetric Components of the Displacement Vector	65
3.4.2	Homogeneous Solutions	66
3.4.3	Solution of Some Problems	71
3.5	Mixed Boundary-Value Problems for an Elastic Half-Plane. Crack Propagation	77
3.5.1	Representations of Solutions to Mixed Boundary-Value Problems	77
3.5.2	Solution for the $n \geq 1$ -Case. Some Examples	79
3.5.3	Solution for the $n \leq 0$ -Case	87
3.6	Solution of Analogous Mixed Boundary-Value Problems. Wedge-Shaped Punch	91
3.6.1	Solution for the $n \geq 1$ -Case	91
3.6.2	Indentation of a Wedge with a Sub-Rayleigh Contact Speed	94
3.6.3	Indentation of a Wedge with Super-Rayleigh Contact Speed	98
3.6.4	Solution Singularities at the Edges of the Contact Region	104
3.7	Interrelation Between Three- and Two-Dimensional Problems ..	106
3.8	Application of the Smirnov–Sobolev Method to Solving Axisymmetric Elastodynamic Problems	112
3.8.1	Representation of Axisymmetric Solutions by Employing Analytical Functions	112
3.8.2	Solutions to Axisymmetric Problems	117
3.9	Solutions to Some Axisymmetric Problems with Mixed Boundary Conditions	120
3.9.1	Solution for the $n \geq 1$ -Case	121
3.9.2	Circular Crack Expansion and a Conical Punch Indentation	123
3.9.3	Expansion of a Circular Crack Due to a Concentrated Load	129
3.10	An Alternative Derivation of the Smirnov–Sobolev Representations	133
4.	Integral Transforms in Elastodynamics	137
4.1	Application of Integral Transforms to Solving Elastodynamic Problems	138
4.2	Lamb's Problem for a Half-Plane	139
4.2.1	Solution to the Problem	140
4.2.2	Cagniard-de Hoop Method	142
4.3	Diffraction of an Acoustic Wave by a Rigid Sphere	149

4.4	Expansion of an Acoustic Wave Solution for a Sphere Over a Time-Dependent Interval	155
4.5	Diffraction of Acoustic Waves by a Rigid Cone	160
4.5.1	Diffraction of a Plane Acoustic Wave by a Rigid Cone	160
4.5.2	Diffraction of a Spherical Acoustic Wave by a Cone ..	168
4.6	Diffraction of Elastic Waves by a Smooth Rigid Cone	173
4.6.1	Diffraction of a Plane Longitudinal Wave by a Cone ..	173
4.6.2	Diffraction of a Spherical Elastic Wave by a Cone	176
4.7	Impact of a Circular Cylinder on a Stationary Obstacle	179
4.7.1	Formulation and Solution of the Problem	179
4.7.2	Analysis of the Solution at the Points of a Cylinder's Axis	183
5.	Solution to Three-Dimensional Elastodynamic Problems with Mixed Boundary Conditions for Wedge-Shaped Domains	190
5.1	Combined Method of Integral Transforms	191
5.1.1	Problem Formulation	191
5.1.2	Problem Solution	192
5.2	Diffraction of a Spherical Elastic Wave by a Smooth Rigid Wedge	204
5.2.1	Solution to the Problem	205
5.2.2	Analysis of the Obtained Solution	210
5.3	Diffraction of an Arbitrary Incident Plane Elastic Wave by a Rigid Smooth Wedge	212
6.	Wiener–Hopf Method in Elastodynamics	220
6.1	Problems with a Stationary Boundary	220
6.1.1	A Semi-Infinite Punch	221
6.1.2	Analysis of the Punch Solution	229
6.2	A Finite-Width Punch	231
6.2.1	Solution to the Problem	231
6.2.2	The Acoustic Case	233
6.3	Problems with Moving Boundary Edges	236
6.3.1	Problem Formulation and Application of Integral Transforms	236
6.3.2	Splitting of Fundamental Solutions	240
6.3.3	Solution of the Two-Dimensional Dynamic Problem ...	243
6.4	Some Crack and Punch Problems	246
6.4.1	The Plane-Strain Problem of a Semi-Infinite Crack Propagation	246
6.4.2	The Antiplane Problem of a Semi-Infinite Crack Propagation	248
6.4.3	The Acoustic Problem for a Punch	254

7. Homogeneous Solutions to Dynamic Problems for Anisotropic Elastic Media (Willis' Method)	256
7.1 Studies in Elastodynamics for Anisotropic Media	256
7.2 Solution to the First Boundary Value Problem	257
7.2.1 Three-Dimensional Case	258
7.2.2 Two-Dimensional Case	269
7.2.3 Radon Transform	271
7.3 Solution to the Second Boundary-Value Problem	272
7.3.1 Three-Dimensional Case	272
7.3.2 Two-Dimensional Case	274
7.4 Lamb's Problem	275
7.4.1 Three-Dimensional Case	276
7.4.2 Two-Dimensional Case	277
7.4.3 Isotropic Half-Space	277
7.5 The Wedge-Shaped Punch Problem	281
7.6 Representing the Solutions for an Anisotropic Space in Terms of Displacement/Stress Discontinuities Across a Plane	288
7.7 Expansion of an Elliptic Crack	294
7.7.1 Solution to the Problem	295
7.7.2 Isotropic Media	298
7.8 Two-Dimensional Problems	300
7.8.1 A Strip-Shaped Crack	301
7.8.2 The Axisymmetric Case	304
References	309
Subject Index	317