

Contents

1	Introduction	1
2	Information Theory	5
2.1	Basic Concepts	5
2.1.1	Information and Shannon Entropy	6
2.1.2	Relative Information, Relative Entropy, or Kullback-Leibler Divergence	10
2.2	Maximum Entropy Formalism	13
2.2.1	Example: The Bose-Einstein Distribution	16
2.3	Generalisation of Entropies to a Continuous State-Space	17
	Problems	18
3	Information Theory for Lattice Systems	21
3.1	Symbol Sequences and Information	21
3.1.1	Probabilistic Description of a Symbol Sequence	23
3.1.2	Quantifying Disorder in a Symbol Sequence	25
3.1.3	Quantifying Order and Correlations in a Symbol Sequence	27
3.2	Markov Processes and Hidden Markov Models	30
3.2.1	Markov Processes and Entropy	30
3.2.2	Hidden Markov Models and Entropy	32
3.2.3	Some Examples	34
3.3	Measuring Complexity	38
3.3.1	Correlation Complexity for Markov Processes and Hidden Markov Models	40
3.4	Extensions to Higher Dimensions	42
	Problems	44
4	Cellular Automata	47
4.1	Cellular Automata—a Class of Discrete Dynamical Systems	47
4.2	Elementary Cellular Automata	50
4.3	Information Theory for Cellular Automata	51

4.3.1	Deterministic Rules	52
4.3.2	Almost Reversible Rules	54
4.3.3	Rules with Noise	56
4.4	Examples of Information-Theoretic Properties in the Evolution of Simple CA	58
4.5	Analysis of CA Time Evolution Using Hidden Markov Models	62
4.6	Local Information Detecting Patterns in CA Time Evolution	65
	Problems	67
5	Physics and Information Theory	73
5.1	Basic Thermodynamics	73
5.1.1	Intensive and Extensive Variables	75
5.2	Work and Information—An Extended Example	76
5.3	From Information Theory to Statistical Mechanics and Thermodynamics	79
5.3.1	Comparing Two Different Gibbs Distributions	81
5.3.2	Information and Free Energy in Non-equilibrium Concentrations	83
5.4	Microscopic and Macroscopic Entropy	84
5.5	Information Theory for Spin Systems	86
5.5.1	Example: The One-Dimensional Ising Model	87
5.6	The Approach Towards Equilibrium in a Reversible One-Dimensional Ising Model	88
5.6.1	Q2R—A Microscopically Reversible Ising Dynamics	89
5.6.2	Analysis of the Time Evolution Starting from a Low Entropy State	90
5.6.3	Time Evolution of the Magnetisation	91
5.6.4	Time Evolution of Information-Theoretic Characteristics	93
	Problems	97
6	Geometric Information Theory	101
6.1	Information Decomposition with Respect to Position and Resolution	101
6.1.1	Resolution-Dependent Probability Density	102
6.1.2	Examples on Resolution-Dependent Probability Densities	104
6.1.3	Connection Between Resolution and Diffusion	105
6.1.4	Decomposition of Information	105
6.2	Dimension, Information, and Fractals	108
6.2.1	Dimensions	108
6.2.2	Fractal Dimension	109
6.2.3	Dimension and Information	110
	Problems	112

- 7 Pattern Formation in Chemical Systems** 115
 - 7.1 The Transition from Micro to Macro 117
 - 7.2 Information Analysis of Chemical Pattern Formation 119
 - 7.2.1 Chemical and Spatial Information 120
 - 7.2.2 Decomposition of Spatial Information in a Chemical Pattern 121
 - 7.2.3 Reaction-Diffusion Dynamics 123
 - 7.2.4 Destruction of Information in Closed Chemical Systems 125
 - 7.2.5 Flows of Information in Closed Chemical Systems 126
 - 7.2.6 A Continuity Equation for Information in the Case of a Closed System 128
 - 7.2.7 A Continuity Equation for Information in the Case of an Open System 129
 - 7.3 Application to the Self-replicating Spots Dynamics 130
- 8 Chaos and Information** 133
 - 8.1 Basic Dynamical Systems Concepts 133
 - 8.1.1 Iterated Maps, Fixed Points, and Periodic Orbits 134
 - 8.1.2 Probability Densities and Measures on the State Space 134
 - 8.1.3 Lyapunov Exponent 135
 - 8.1.4 The Lyapunov Exponent as an Information Flow from “Micro” to “Macro” 136
 - 8.2 Dynamical Systems Entropy and Information Flow 137
 - 8.2.1 Extended Example of a Generating Partition for a Skew Roof Map 139
 - 8.2.2 An Example of a Partition that is not Generating 142
 - Problems 144
- 9 Appendix** 149
- References** 151